MONTESSORI CURRICULUM TO STANDARDS ALIGNMENT

ELEMENTARY • 1ST–6TH GRADE

MATHEMATICS

NATIONAL CENTER for
MONTESSORI in the PUBLIC SECTOR

Montessori Curriculum to Standards Alignment
Elementary • 1st–6th Grade
Mathematics

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Assessment vocabulary drawn from Marzano Resources free online resource, Basic Vocabulary Terms (marzanoresources.com/media/documents/reproducibles/vocab-common-core/basic-terms-and-phrases.pdf)

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CHAPTER 1

MATHEMATICAL PRACTICE

HUMAN TENDENCIES AND THE MATHEMATICAL MIND

MATHEMATICAL PRACTICE AND MONTESSORI

CCSS Standards for Mathematical Practice

The Common Core State Standards for Mathematics begin with eight Standards for Mathematical Practice (CCSS.MATH.PRACTICE.MP1-8): "processes and proficiencies," rather than specific grade-level skills and concepts, which mathematics educators at all levels should seek to develop in their students.

These Standards are typically grouped into four interrelated clusters:

- Problem Solving
  - MP1: Make sense of problems and persevere in solving them.
  - MP6: Attend to precision.
- Communication: Reasoning and Explaining
  - MP2: Reason abstractly and quantitatively.
  - MP3: Construct viable arguments and critique the reasoning of others.
- Modeling and Using Tools
  - MP4: Model with mathematics
  - MP5: Use appropriate tools strategically.
- Seeing Structures and Generalizing
  - MP7: Look for and make use of structure.
  - MP8: Look for and express regularity in repeated reasoning.

Montessori Mathematics and Geometry

Montessori lessons which support the development of these practices are found in every section of the mathematics and geometry curriculum. The Montessori approach is grounded in a model of human development which explicitly addresses these standards with two concepts: the Human Tendencies and the Mathematical Mind. The Montessori mathematics manipulatives (the "Montessori Materials") also support the development of these processes and proficiencies through didactic presentation by adults and independent work by children.

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MATHEMATICAL PRACTICE AND MONTESSORI

Human Tendencies

Montessori recognizes basic human behaviors and inclinations and behaviors termed "Human Tendencies", elaborated in The Human Tendencies and Montessori Education by (Montessori, Mario, Association Montessori Internationale, 1956). They are generally summarized as:

- **Orientation:** Observing one's environment and building an ordered mental model of the relevant elements.
- **Order:** Closely related to orientation, the tendency to build an ordered mental model and place material objects, entities, and concepts within it.
- **Movement:** Physical movement of the body or parts of the body to interact with the environment.
- **Exploration:** Physical and mental interaction with the environment and with mental models to better understand and develop their qualities.
- **Abstraction/Imagination/Reason:** Thinking about things not physically present, categorizing, reasoning, and creating new ideas.
- **Manipulation:** Using the hands or the body to reorder or change elements of the environment.
- **Work:** Persistent activity to achieve a desired end.
- **Exactness:** Awareness of the possibility for precision, and activity directed at achieving it.
- **Repetition:** Repetition, possibly with variation, of an activity or sequence, often with the goal of improving effectiveness.
- **Self-Perfection:** Activity specifically directed at improving and refining practice.
- **Communication:** Exchanging information with other humans.

Mathematical Mind

In the Montessori framework, the Human Tendencies operate throughout development and across the entire range of human experience. Montessori also recognizes an innate disposition towards mathematics as the "Mathematical Mind", a disposition which is present in all human beings but which must be supported and cultivated. The characteristics of the Mathematical Mind (which overlap with the Human Tendencies) are:

- A propensity to learn things which enhance the ability to be **exact** and **orderly**, and to **observe**, **compare**, and **classify**.
- Tendencies to **calculate**, **measure**, **reason**, **abstract**, **imagine,** and **create**.

Montessori Materials

The Montessori approach incorporates manipulative materials for mathematics (and other subjects) designed to introduce and reinforce mathematical and geometric concepts through direct instruction and independent exploration. The materials share some common characteristics:

- Attractive and engaging appearance that invites independent exploration.
- Mathematically precise construction.
- Appealing to multiple senses (e.g., touch color, size, shape, etc.)
- Isolation of a single concept.
- Material representation of abstract concepts.

NOTES

References and Resources:

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- Montessori, Mario. (1966). *The Human Tendencies and Montessori Educawtion.* Association Montessori Internationale
- O'Connell, S., & SanGiovanni, J. (2013). Putting the Practices Into Action: Implementing the Common Core Standards for Mathematical Practice, K-8 (Illustrated ed.). Heinemann.
- *Standards for Mathematical Practice | Common Core State Standards Initiative*. (n.d.). Common Core State Standards Initiative. Retrieved April 15, 2021, from http://www.corestandards.org/Math/Practice/

PROBLEM SOLVING

SKILLS INVENTORY

Lower and Upper Elementary

Solve problems in all areas of the Montessori math curriculum, attending to precision in use of the materials, communication, and calculations, and persevering by trying different approaches and seeking support.

MONTESSORI PRINCIPLES AND SPECIFIC SKILLS

Human Tendencies

Human Tendencies relevant to Problem Solving include:

- Order
- Exactness
- Abstraction/Imagination/Reason
- Work

Mathematical Mind

Characteristics of the Mathematical Mind to be supported In this area include:

- Order
- Exactness
- Calculation
- Measurement

Specific skills to develop for Problem Solving include:

Perseverance

- Try multiple strategies if one does not work.
- Seek support from peers or an adult if a problem is unfamiliar or challenging.
- Reflect on similar math experiences when faced with something new.
- Try to visualize the problem and the solution.

Precision

- Be precise:
  - In use of the materials.
  - In communication so others understand what they mean.
  - In calculations.
  - In explanations of what the work or data they are presenting means.
- Use the precise language of mathematics:
  - Use terms accurately.
  - Label work with care.

ASSESSMENT CONSIDERATIONS

INITIAL SERIES

Students will be asked to:

- Realize that doing mathematics involves solving problems.
- Discuss how they solved a problem.
- Explain the meaning of a problem and look for ways to solve it.
- Use Montessori materials or pictures to help them conceptualize and solve problems.
- Try other approaches to solve problems.
- Make conjectures about the solution.
- Plan out a problem solving approach.
- Check their thinking by asking themselves, "Does this make sense?"
- Develop mathematical communication skills.
- Use clear and precise language in their discussions with others and when they explain their own reasoning.

MIDDLE SERIES

Students will be asked to:

- Demonstrate that mathematics involves solving problems.
- Listen to the strategies of others and try different approaches.
- Use another method to check their answers.
- Check their thinking by asking themselves, "Does this make sense?"
- Specify units of measure and state the meaning of the symbols they choose. (Example: When finding the area of a rectangle they record their answers in square units.)

LATER SERIES

Students will be asked to:

- Seek the meaning of a problem and look for efficient ways to represent and solve it.
- Solve real world problems through the application of algebraic and geometric concepts.
- Seek the meaning of a problem and look for efficient ways to represent and solve it.
- Check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"
- Use appropriate terminology when referring to expressions, fractions, geometric figures, and coordinate grids.
- Using clear and precise language in their discussions with others and in their own reasoning.

COMMON CORE STATE STANDARDS (CCSS.MATH.PRACTICE)

STANDARDS FOR MATHEMATICAL PRACTICE (MP)

MP1	Make sense of problems and persevere in solving them.
MP6	Attend to precision.

NOTES

COMMUNICATION: REASONING AND EXPLAINING

SKILLS INVENTORY

Lower and Upper Elementary

Uses communication in all areas of the Montessori math curriculum by constructing viable arguments or critiquing the reasoning of others through collaboration, use of vocabulary (Montessori, academic, domain-specific) and reasoning with the Montessori materials and relating to real-world applications.

MONTESSORI PRINCIPLES AND SPECIFIC SKILLS

Human Tendencies

Human Tendencies relevant to Communication: Reasoning and Explaining include:

- Abstraction/Imagination/Reason
- Communication

Mathematical Mind

Characteristics of the Mathematical Mind to be supported In this area include

- Reasoning
- Abstraction

Specific skills to develop for Communication: Reasoning and Explaining include:

Reasoning

- Use the Montessori materials or models to explain thinking.
- Explain the numbers and symbols that are being used.
- Relate thinking to a real-world application.

Vocabulary

- Know the correct Montessori, academic and domain-specific vocabulary.

Collaboration

- Work with classmates on math.

Communication

- Make and defend a mathematical point.
- Justify conclusions by communicating what they think and why they think it.
- Listen and follow other students' reasoning and say if the reasoning makes sense or leaves them with questions.

ASSESSMENT CONSIDERATIONS

INITIAL SERIES

Students will be asked to:

- Recognize that a number represents a specific quantity.
- Connect the quantity to written symbols.
- Create representations of a problem showing understanding of the quantities.
- Know and use different properties of operations and objects.
- Construct arguments using objects, pictures, drawings, and actions.
- Listen to others' explanations, ask questions and decide if the explanations make sense.
- Practice mathematical communication skills in discussions involving questions like "How did you get that?", "Explain your thinking" and "Why is that true?"

MIDDLE SERIES

Students will be asked to:

- Create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities.
- Write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts.
- Explain their thinking to others and respond to others' thinking.

LATER SERIES

Students will be asked to:

- Create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities.
- Represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities.
- Contextualize to understand the meaning of the number or variable as related to the problem.
- Decontextualize to manipulate symbolic representations by applying properties of operations.
- Explain calculations based upon models and properties of operations and rules that generate patterns.
- Construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays.
- Refine mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students.
- Pose questions like "How did you get that?", "Why is that true?", and "Does that always work?"

COMMON CORE STATE STANDARDS (CCSS.MATH.PRACTICE)

STANDARDS FOR MATHEMATICAL PRACTICE

MP2	Reason abstractly and quantitatively.
MP3	Construct viable arguments and critique the reasoning of others.

NOTES

MODELING AND USING TOOLS

SKILLS INVENTORY

Lower and Upper Elementary

Uses Montessori materials in all areas of the Montessori math curriculum including choosing the appropriate Montessori material or model (drawing, table, chart, graph) to set up math equations or solve real-life problems.

MONTESSORI LESSONS

Human Tendencies

Aspects of the Human Tendencies relevant to Modeling and Using Tools include the tendencies for

- Abstraction/Imagination/Reason
- Manipulation

Mathematical Mind

Characteristics of the Mathematical Mind to be supported In this area include tendencies to:

- Measure
- Reason
- Abstract
- Imagine

Specific activities to support Modeling and Using Tools include:

Montessori Materials

- Use Montessori materials to find solutions.
- As students move towards abstraction, use drawings, tables, charts, graphs, words to find solutions.
- Consider the most efficient tool for the task.

Real-life situations

- Apply math to real-life situations.
- Set up a math problem or equation with Montessori materials or on paper.
- Return to the question and explain their answer in relationship to the situation.

ASSESSMENT CONSIDERATIONS

INITIAL SERIES

Students will be asked to:

- Experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, creating equations, etc.
- Connect the different representations and explain the connections.
- Decide which Montessori material will help solve the problem.
- Decide which tool might be better suited.
- Solve a problem by drawing a picture rather than writing an equation.

MIDDLE SERIES

Students will be asked to:

- Connect the different representations and explain the connections.
- Evaluate their results in the context of the situation.
- Reflect on whether the results make sense.
- Use graph paper to find solve a problem.
- Compile the possibilities into an organized list or a table.
- Use measurement tools to understand the relative size of units within a system.
- Express measurements given in larger units in terms of smaller units.

LATER SERIES

Students will be asked to:

- Evaluate the utility of models to determine which models are most useful and efficient to solve problems.
- Model problem situations symbolically, graphically, tabularly, and contextually.
- Form expressions, equations, or inequalities from real world contexts and connect symbolic and graphical representations.
- Explore covariance and represent two quantities simultaneously.
- Use number lines to compare numbers and represent inequalities.
- Use measures of center and variability and data displays (i.e., box plots and histograms) to draw inferences about and make comparisons between data sets.
- Connect and explain the connections between the different representations.
- Use all representations as appropriate to a problem context.
- Solve problems or make predictions from real world data.
- Decide to represent similar data sets using dot plots with the same scale to visually compare the center and variability of the data.
- Use physical objects or applets to construct nets and calculate the surface area of three-dimensional figures.

COMMON CORE STATE STANDARDS (CCSS.MATH.PRACTICE)

STANDARDS FOR MATHEMATICAL PRACTICE

MP4	Model with mathematics.
MP5	Use appropriate tools strategically.

NOTES

SEEING STRUCTURES AND GENERALIZING

SKILLS INVENTORY

Lower and Upper Elementary

Looks for and recognizes patterns in all areas of the Montessori math curriculum and reasons about mathematical structures.

MONTESSORI LESSONS

Human Tendencies

Aspects of the Human Tendencies relevant to Seeing Structures and Generalizing include the tendencies for

- Order
- Abstraction/Imagination/Reason

Mathematical Mind

Characteristics of the Mathematical Mind to be supported In this area include tendencies for:

- Reason
- Abstraction

Specific skills to develop for Seeing Structures and Generalizing include:

Mathematical Structures

- Mathematical structures, including
  - The number sequence.
  - The place value system.
  - Properties of operations.
  - Formulas for finding area.
  - Inverse relationship of addition and subtraction.
  - Inverse relationship of multiplication and division.
- Recognize similarities and patterns that emerge through repetition.
- Look closely for a pattern or structure.

ASSESSMENT CONSIDERATIONS

INITIAL SERIES

Students will be asked to:

- Discern a pattern or structure. (example: if students recognize 12 + 3 = 15, then they also know 3 + 12 = 15 (Commutative property of addition). To add 4 + 6 + 4, the first two numbers can be added to make a ten, so 4 + 6 + 4 = 10 + 4 = 14).
- Look for patterns. (Example: Adopting mental math strategies based on patterns—making ten, fact families, doubles).
- Notice repetitive actions in counting and computation, etc. When children have multiple opportunities to add and subtract "ten" and multiples of "ten" they notice the pattern and gain a better understanding of place value.
- Check their work by asking themselves, "Does this make sense?"

MIDDLE SERIES

Students will be asked to:

- Look closely to discover a pattern or structure.
- Use properties of operations to explain calculations.
- Relate representations of counting problems such as tree diagrams and arrays to the multiplication principal of counting.
- Generate number or shape patterns that follow a given rule.
- Notice repetitive actions in computation and look for more shortcut methods.
- Use the distributive property as a strategy for using products they know to solve products that they don't know.
- Continually evaluate their work by asking themselves, "Does this make sense?"
- Notice repetitive actions in computation to make generalizations.
- Use models to explain calculations, examine patterns and generate their own algorithms.

continues

ASSESSMENT CONSIDERATIONS

LATER SERIES

Students will be asked to:

- Use properties of operations as strategies to add, subtract, multiply and divide with whole numbers, fractions, and decimals.
- Examine numerical patterns and relate them to a rule or a graphical representation.
- Seek patterns or structures to model and solve problems.
- Recognize patterns that exist in ratio tables recognizing both the additive and multiplicative properties.
- Apply properties to generate equivalent expressions (i.e., 6 + 2x = 3 (2 + x) by distributive property).
- Solve equations (i.e., 2c + 3 = 15, 2c = 12 by subtraction property of equality, c = 6 by division property of equality).
- Compose and decompose two- and three- dimensional figures to solve real world problems involving area and volume.
- Use repeated reasoning to understand algorithms and make generalizations about patterns.
- Connect place value and their prior work with operations to understand algorithms to fluently multiply multi-digit numbers.
- Perform all operations with decimals to hundredths.
- Explore operations with fractions with visual models and begin to formulate generalizations.
- Use repeated reasoning to understand algorithms and make generalizations about patterns.
- Solve and model problems. They may notice that a/b ÷ c/d = ad/bc and construct other examples and models that confirm their generalization.
- Connect place value and their prior work with operations to understand algorithms to fluently divide multi-digit numbers and perform all operations with multi-digit decimals.
- Informally begin to make connections between covariance, rates, and representations showing the relationships between quantities.

COMMON CORE STATE STANDARDS (CCSS.MATH.PRACTICE)

STANDARDS FOR MATHEMATICAL PRACTICE

MP7	Look for and make use of structure.
MP8	Look for and express regularity in repeated reasoning.

NOTES

CHAPTER 2

FOUNDATIONS

GREAT STORY: THE STORY OF NUMBERS

SKILLS INVENTORY

Lower Elementary

Listens to stories about significant historical changes and is inspired to gather additional information to clarify or deepen understanding.

MONTESSORI LESSONS	PURPOSES
INITIAL SERIES	
Great Story: The Story of Numbers	• To generate questions about individuals and groups who have shaped a significant historical change. • To inspire children to ask and answer questions about information that has been presented orally. • To inspire children to gather additional information to clarify comprehension or deepen understanding. • To give the child the idea of numbers and expose the child to the history with our numerals. • To inspire appreciation of the unknown heroes who created our numeral system. • To help children recognize that numbers help humans meet their needs.

ASSESSMENT VOCABULARY

INITIAL SERIES

answer	speaker	**Cognitive Verbs**
ask	text	answer
detail	topic	ask
information		clarify
issue		describe
media/medium		present
question		recount

ASSESSMENT CONSIDERATIONS

Students will not be assessed on the contents of the Great Stories.

COLLEGE, CAREER AND CIVIC LIFE (C3) FRAMEWORK FOR STATE SOCIAL STUDIES STANDARDS

HISTORY (D2.HIS)

CHANGE, CONTINUITY AND CONTEXT

His.3.K-2	Generate questions about individuals and groups who have shaped a significant historical change.
His.3.3-5	Generate questions about individuals and groups who have shaped significant historical changes and continuities.

COMMON CORE STATE STANDARDS (CCSS.ELA-LITERACY)

LANGUAGE: SPEAKING AND LISTENING (SL)

COMPREHENSION AND COLLABORATION

SL.1.2	Ask and answer questions about key details in a text read aloud or information presented orally or through other media.
SL.1.3	Ask and answer questions about what a speaker says in order to gather additional information or clarify something that is not understood.
SL.2.2	Recount or describe key ideas or details from a text read aloud or information presented orally or through other media.
SL.2.3	Ask and answer questions about what a speaker says in order to clarify comprehension, gather additional information, or deepen understanding of a topic or issue.

NOTES

CHAPTER 3

THE DECIMAL SYSTEM

NUMBER AND QUANTITY

SKILLS INVENTORY
Lower Elementary Understands one-to-one correspondence and number symbol, reads and writes whole numbers using numerals, words and expanded form and can identify odd and even numbers.

MONTESSORI LESSONS	PURPOSES
INITIAL SERIES	
NUMBERS 1 TO 10	
Cards and Counters **Memory Games of Numbers** **Colored Bead Bars** • Bead Stair • Snake Game—In Search of 10	• To reinforce the knowledge that each number is made up of separate quantities. • To verify the sequence of numbers from 1-10. • To verify the quantities that correspond to each symbol. • To introduce the quantities and symbols for numbers 0-10. • To develop the power of memory to hold a numeric symbol in the mind. • To help the child transfer knowledge of the numbers from 0-10 from the specific materials to the objects of daily life.

continues

MONTESSORI LESSONS	PURPOSES
COUNTING AND NUMBERS GREATER THAN 10	
Golden Beads • Introduction to Quantity • Games with Quantities • Change Game • Knowledge of the Quantities • Knowledge of the Symbols • Quantity and Symbol • Exchange Game • Reverse Exchange Game	• To introduce the names for quantities in each category ("unit"; "ten"; "hundred"; "thousand"). • To show the relationship between one category and the next. • To offer the child the sensorial experience of the relative sizes of the categories (bulk). • To extend the sensorial experience of the different categories and the difference in bulk, for instance, between 6 units and 6 hundreds. • To reinforce language of the categories. • To introduce the symbols for multiple 1,000s, 100s, 10s, 1s. • To associate the concrete representation and symbolic representation of the categories. • To read numbers to 1,000 using number names. • To write numbers to 1,000 using base-ten numerals. • To ensure the child knows that zero can hold a place for a category. • To count forward starting at any number besides 0.
Number Cards • Introduction to Symbol • Games with Symbols	• To read numbers to 1,000 using number names. • To write numbers to 1,000 using base-ten numerals. • To ensure the child knows that zero can hold a place for a category.
Seguin Boards: Teens • Introduction to Quantity • Games with Quantities • Introduction to Symbols • Games with Symbols • Association of Quantity and Symbol	• To introduce the quantities 11-19 with their names. • To see the relationship of the numbers 1-9 to the number 10. • To give the symbols for the numbers 11-19. • To help the child become secure with the symbols for the numbers 11-19. • Association of the quantity, name, and symbol for the teens (11-19).
Seguin Boards: Tens • Naming the Tens • Counting from 10-99	• To introduce the conventional names for the tens quantities: "twenty", "thirty", "forty"…"ninety". • To realize how the numbers progress from one ten to the next. • To connect name, quantity, and symbol for the numbers 11-99. • To realize how the numbers progress from one ten to the next.

continues

MONTESSORI LESSONS	PURPOSES
Bead Cabinet • 100 Chain and Short Chains • 1000 Chain and Long Chains • Recording the Chains • Number Pyramid	• To consolidate linear counting. • To count, read and write numerals and represent a number of objects with a written numeral. • To solidify the idea that 100 is 10 tens. • To give a sensorial impression of the difference between 102 and 103. • To solidify the idea that 100 is 10 tens, and 10 hundreds is 1,000. • Comparison of the squares of the numbers 1-10. • To give the child another means of counting in a series. • Count within 1,000; skip-count by 5s, 10s, and 100s. • Motivation to repeatedly count the long and short chains. • To read numbers to 1,000 using number names. • To write numbers to 1,000 using base-ten numerals. • To count forward starting at any number besides 0.
Odd and Even • Cards and Counters • Hundred Board	• To determine whether a group of objects has an odd or even number of members. • To write an equation to express an even number as a sum of two equal addends.
Expanded Form • Teacher-Created Lesson	• To read and write numbers to 1,000 using expanded form.
10 more or 10 Less • Teacher-Created Lesson	• To mentally find 10 more or 10 less of when given a two-digit number without having to count.

ASSESSMENT VOCABULARY

INITIAL SERIES

addend
base-ten numeral
count
count by 100s
count by 10s
count by 2s
count by 5s
equal
equation
even number
expanded form
group/grouping
less than (<)
number
number name
numeral
odd number
pair
range (of numbers or data)
represent
sum
twenty
two

Cognitive Verbs
determine
express
represent

ASSESSMENT CONSIDERATIONS

INITIAL SERIES

Students will be asked to:

Numbers 1 to 10

- Read and write numbers to show how many objects are in a group (up to 120). (1.NBT.A.1)

Counting and Numbers Greater than 10

- Count up to 120 starting at any number under 120. (1.NBT.A.1)
- Read and write numbers to show how many objects are in a group (up to 120). (1.NBT.A.1)
- Mentally find 10 more or 10 less. (1.NBT.C.5)
- Count to 1,000 by 1s, 5s, 10s and 100s. (2.NBT.A.2)
- Read and write numbers to 1,000 in different ways. (2.NBT.A.3)

Odd and Even

- Group objects to tell if a number is odd or even. (2.OA.C.3)
- Write a number sentence to show how adding two of the same number will equal an even number. (2.OA.C.3)

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)

NUMBER AND OPERATIONS IN BASE TEN (NBT)

EXTEND THE COUNTING SEQUENCE	
1.NBT.A.1	Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.
UNDERSTANDING PLACE VALUE	
2.NBT.A.2	Count within 1000; skip-count by 5s, 10s, and 100s.
2.NBT.A.3	Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
USE PLACE VALUE UNDERSTANDING AND PROPERTIES OF OPERATIONS TO ADD AND SUBTRACT	
1.NBT.C.5	Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

continued

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)

OPERATIONS AND ALGEBRAIC THINKING (OA)

WORK WITH EQUAL GROUPS OF OBJECTS TO GAIN FOUNDATIONS FOR MULTIPLICATION

2.OA.C.3	Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

NOTES

PLACE VALUE

SKILLS INVENTORY

Lower Elementary

Uses knowledge of place value to explain the values of digits in a number, to read and write numbers using base-ten numerals, number names, and expanded form, to round a number and to compare two numbers using greater than, less than and equal to.

Upper Elementary

Uses knowledge of place value to explain the values of digits in a larger number, to read and write multi-digit numbers using base-ten numerals, number names, and expanded form, to round a number, to compare two numbers using greater than, less than and equal to and recognizes that a digit in one place is 10 times more (right) or 1/10 less (left) than its neighboring digit.

MONTESSORI LESSONS	PURPOSES
INITIAL SERIES	
THE DECIMAL SYSTEM, CATEGORIES AND NUMERATION	

Golden Beads and Cards
• Association of Quantity and Symbol
  • Bead to Card
  • Card to Bead
• Formation of Numbers
• Formation of Numbers with Zero
• Tray of Nine
	• To introduce, reinforce and extend the concept of families in the decimal system (simple, tens, hundreds).
	• To help the child associate symbol, name, and quantity of the decimal categories.
	• To understand that 10 can be thought of as a bundle of ten ones—called a "ten."
	• To understand that 100 can be thought of as a bundle of ten tens—called a "hundred."

Wooden Hierarchical Material • Quantity and Language • Geometric Layout • Introduction to Symbol • Symbol and Quantity	• To experience quantity and language for hierarchical numbers larger than thousands. • To review and extend the names of the decimal system categories: one, ten, hundred, one thousand, ten thousand, hundred thousand, one million. • To introduce, reinforce and extend the concept of families in the decimal system (simple, tens, hundreds). • To help the child associate symbol, name, and quantity of the decimal categories. • To understand that 10 can be thought of as a bundle of ten ones—called a "ten." • To understand that 100 can be thought of as a bundle of ten tens—called a "hundred."

continues

MONTESSORI LESSONS	PURPOSES
Hierarchical Frames • Introduction to the Bead Frame • Reading Numbers • Introduction to 0 • Composing and Writing Large Numbers	• To become familiar with the decimal system. • To learn to read multi-digit numbers. • Read and write numbers to 1,000 using base-ten numerals, number names, and expanded form. • For the children to see that a numeral's position tells us its category value. • To consolidate the understanding that each position has a category value. • To understand that the two digits of a two-digit number represent amounts of tens and ones. • To understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones. • To understand that the numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. • To understand that the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). • To understand that the numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
Greater Than, Less Than, Equal To • Teacher-Created Lessons	• To compare two numbers using place value knowledge. • To use the symbols >, = and < to show the comparison. • To compare two two-digit numbers based on meanings of the tens and ones digits.
Reading and Writing Numbers • Teacher-Created Lesson	• To read and write numbers using base-ten numerals. • To read and write numbers using number name.
Expanded Form • Teacher-Created Lesson	• To read and write numbers using expanded form.

continues

MONTESSORI LESSONS	PURPOSES
MIDDLE SERIES	
Rounding • Stamp Game • Number Line	• To mentally find 10 more or 10 less than the number. • Use place value understanding to round multi-digit whole numbers to any place. • Use place value understanding to round whole numbers to the nearest 10 or 100.
Greater Than, Less Than, Equal To • Teacher-Created Lessons	• To compare two multi-digit numbers using place value knowledge. • To use the symbols >, = and < to show the comparison.
Reading and Writing Numbers • Teacher-Created Lesson	• To read and write multi-digit whole numbers using base-ten numerals. • To read and write multi-digit whole numbers using number names.
Expanded Form • Teacher-Created Lesson	• To read and write multi-digit whole numbers using expanded form.
10 times More • Teacher-Created Lesson	• To recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.
LATER SERIES	
10 times More and 1/10 Less • Teacher-Created Lessons	• To recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. • To recognize that in a multi-digit number, a digit in one place represents 1/10 of what it represents in the place to its left.

ASSESSMENT VOCABULARY

INITIAL SERIES	MIDDLE SERIES	LATER SERIES
amount base-ten numeral compare comparison count digit expanded form hundred hundreds less mental math more number number number name one ones represent symbol ten tens three three-digit number two two-digit number **Cognitive Verbs** compare explain record represent understand	*In addition to previous vocabulary:* base-ten numeral compare comparison digit expanded form multi digit number number name place place value represent rounding symbol ten two whole number **Cognitive Verbs** recognize	*In addition to previous vocabulary:* one tenths

ASSESSMENT CONSIDERATIONS

INITIAL SERIES

Students will be asked to:

The Decimal System, Categories and Numeration

- Tell how many tens and how many ones are in a number. (1.NBT.B.2)
- Show understanding of what a "ten" is. (1.NBT.B.2.A)
- Show that any number between 11 and 19 is a group of "ten" and a certain number of ones. (1.NBT.B.2.B)
- Demonstrate understanding that the numbers used when counting by tens, have a certain number of tens and 0 ones. (1.NBT.B.2.C)
- Understand and use hundreds, tens and ones. (2.NBT.A.1)
- Demonstrate understanding that a bundle of ten "tens" is called a "hundred". (2.NBT.A.1.A)
- Demonstrate understanding that the numbers used when counting by hundreds, have a certain number of hundreds, 0 tens and 0 ones. (2.NBT.A.1.B)
- Read and write numbers to 1,000 in different ways. (2.NBT.A.3)

Greater Than, Less Than, Equal To

- Demonstrate understanding of tens and ones by comparing two-digit numbers using <, =, and >. (1.NBT.B.3)
- Demonstrate understanding of hundreds, tens and ones by comparing three-digit numbers using <, =, and >. (2.NBT.A.4)

MIDDLE SERIES

Students will be asked to:

The Decimal System, Categories and Numeration

- Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. (4.NBT.A.1)

Greater Than, Less Than, Equal To

- Compare two larger numbers using knowledge about the values in each place. (4.NBT.A.2)
- Compare two larger numbers and use the symbols >, = and < to show the comparison. (4.NBT.A.2)

Rounding

- Use place value to round numbers to the nearest 10 or 100. (3.NBT.A.1)
- Round larger whole numbers to any place. (4.NBT.A.3)

Reading and Writing Numbers

- Read and write larger whole numbers using numerals, words and in expanded form. (4.NBT.A.2)

LATER SERIES

Students will be asked to:

The Decimal System, Categories and Numeration

- Recognize that a digit in one place represents 10 times as much as it represents in the place to its right. (5.NBT.A.1)
- Recognize that a digit in one place represents 1/10 as much as it represents in the place to its left. (5.NBT.A.1)

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)	
NUMBER AND OPERATIONS IN BASE TEN (NBT)	
UNDERSTAND PLACE VALUE	
1.NBT.B.2	Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:
1.NBT.B.2.A	10 can be thought of as a bundle of ten ones—called a "ten".
1.NBT.B.2.B	The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
1.NBT.B.2.C	The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).
1.NBT.B.3	Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.
2.NBT.A.1	Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
2.NBT.A.1.A	100 can be thought of as a bundle of ten tens—called a "hundred".
2.NBT.A.1.B	The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
2.NBT.A.3	Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
2.NBT.A.4	Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.
USE PLACE VALUE UNDERSTANDING AND PROPERTIES OF OPERATIONS TO PERFORM MULTI-DIGIT ARITHMETIC	
3.NBT.A.1	Use place value understanding to round whole numbers to the nearest 10 or 100.

continues

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)

GENERALIZE PLACE VALUE UNDERSTANDING FOR MULTI-DIGIT WHOLE NUMBERS	
4.NBT.A.1	Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.
4.NBT.A.2	Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
4.NBT.A.3	Use place value understanding to round multi-digit whole numbers to any place.
UNDERSTAND THE PLACE VALUE SYSTEM	
5.NBT.A.1	Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

NOTES

CHAPTER 4

WHOLE NUMBERS

OPERATIONS: ADDITION

SKILLS INVENTORY

Uses vocabulary for talking about addition (first addend, second addend, sum), subtraction (minuend, subtrahend, difference), multiplication (multiplicand, multiplier, product) and division (dividend, divisor, quotient).

Lower Elementary

Solves whole number mathematical equations and word-problems using all four operations with Montessori materials (to the materials full place value capacity) moving towards abstraction.

Upper Elementary

Solves whole number mathematical equations and word-problems using all four operations abstractly.

MONTESSORI LESSONS	PURPOSES
INITIAL SERIES	
OPERATIONS	
Golden Beads • Static • Dynamic	• To use Montessori materials, concrete models, and drawings to solve addition equations. • To solve addition equations using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. • To explain why addition strategies work, using place value and the properties of operations. • To give the sensorial impression of addition: Putting quantities together to form a larger quantity. • To understand that in adding multi-digit numbers, one adds ones and ones, tens and tens, hundreds and hundreds etc. • To understand that in adding multi-digit numbers, sometimes it is necessary to compose a ten, hundred, thousand, etc. • To add up to four multi-digit numbers using strategies based on place value and properties of operations.

continues

MONTESSORI LESSONS	PURPOSES
Bead Bars • Addition • Addition with Multiple Addends	• To use Montessori materials, concrete models, and drawings to solve addition equations. • To solve addition equations using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. • To explain why addition strategies work, using place value and the properties of operations. • To give the sensorial impression of addition: Putting quantities together to form a larger quantity. • Relate counting to addition. • To understand that in adding multi-digit numbers, sometimes it is necessary to compose a ten, hundred, thousand, etc.
Stamp Game • Introduction to the Material • Addition	• To use Montessori materials, concrete models, and drawings to solve addition equations. • To solve addition equations using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. • To explain why addition strategies work, using place value and the properties of operations. • To apply the symbols for the operations: + - x ÷ to the decimal system. • To understand the meaning of the equal sign and determine if equations involving addition are true or false. • To understand that in adding multi-digit numbers, one adds ones and ones, tens and tens, hundreds and hundreds etc. • To understand that in adding multi-digit numbers, sometimes it is necessary to compose a ten, hundred, thousand etc. • To add up to four multi-digit numbers using strategies based on place value and properties of operations.
Dot Game • One Number at a Time • Column Addition	• To use Montessori materials, concrete models, and drawings to solve addition equations. • To solve addition equations using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. • To explain why addition strategies work, using place value and the properties of operations. • To focus on the mechanism of carrying numbers up to the next category. • To reinforce the relationship of one category to another. • To understand that in adding multi-digit numbers, one adds ones and ones, tens and tens, hundreds and hundreds, etc. • To understand that in adding multi-digit numbers, sometimes it is necessary to compose a ten, hundred, thousand, etc.

continues

MONTESSORI LESSONS	PURPOSES
Hierarchical Frames • Static • Dynamic	• To provide the child with an opportunity to work in a more symbolic way with addition. • To offer an opportunity for the child to apply memorized knowledge of the essential combinations. • To reinforce that there are not more than 9 in any category in the decimal system. • To reinforce zero as a place holder. • To offer an opportunity to practice recording quantities. • To allow the child to eventually understand the algorithms for addition. • To understand that in adding multi-digit numbers, one subtracts ones and ones, tens and tens, hundreds and hundreds, etc. • To understand that in adding multi-digit numbers, sometimes it is necessary to compose, decompose a ten, hundred, thousand, etc. • To relate counting to addition.
Unknown Number • Teacher-Created Lessons	• To solve an addition equation with unknowns in all positions. • To determine the unknown whole number in an addition equation relating three whole numbers.
Problem Solving • Teacher-Created or Purchased Cards • Experiences in the Classroom	• To verify the concepts of the operations of the decimal system. • To offer an opportunity to apply the knowledge of the operations to real-life situations. • To use addition to solve word problems involving situations of adding to, putting together, and comparing, with unknowns in all positions. • To solve word problems that call for addition of three whole numbers. • To use addition and to solve one- and two-step word problems involving situations of adding to, putting together, and comparing, with unknowns in all positions.
Array • Teacher-Created Lessons	• To use addition to find the total number of objects arranged in rectangular arrays. • To write an equation to show the total number of objects that are in an array.

continues

MONTESSORI LESSONS	PURPOSES
MIDDLE SERIES	
Problem Solving • Teacher-Created or Purchased Cards • Experiences in the Classroom	• To solve multi-step word problems involving whole numbers using knowledge of addition. • To solve multi-step word problems by using equations with a letter standing for the unknown number.
Patterns • Teacher-Created Lessons	• To find patterns of addition. • To create patterns of addition following a given rule. • To generate a number or shape pattern that follows a given rule. • To generate two numerical patterns using two given rules.
LATER SERIES	
Patterns • Teacher-Created Lessons	• To find patterns in addition tables and explain them using knowledge about how numbers work. • To create a number or shape pattern that follows a given rule. • To notice and point out different features of a pattern once it is created by a rule.

ASSESSMENT VOCABULARY

INITIAL SERIES

add/ addition
addend
array
column
compare
compose
compose a ten
concrete
count
decompose/ decomposition
equal
equation
false
hundreds
less than (<)
multiple of ten
number
one-digit number
ones
one-step problem
place value
properties of operations
put together
rectangular array
relationship
represent
row
strategy
subtract/ subtraction
sum symbol
take apart
take from
tens
three-digit number
total
true
two-digit number
unknown
whole number
word problem

Cognitive Verbs

arrange
compare
compose
decompose
explain
express
relate
represent
solve
understand

MIDDLE SERIES

In addition to previous vocabulary:

addition table
equation
estimation
mental math
multiplication table
multi step problem
operation
pattern
properties of operations
quantity
reasonable
remainder
represent
rounding
rule
strategy
two-step problem
unknown
whole number
word problem

Cognitive Verbs

answer
assess
explain
generate
identify
interpret
pose
represent
solve

LATER SERIES

In addition to previous vocabulary:

coordinate plane
corresponding terms
graph
ordered pair
pattern
relationship
rule

Cognitive Verbs

form
generate
graph

ASSESSMENT CONSIDERATIONS

INITIAL SERIES

Students will be asked to:

Operations

- Demonstrate understanding of how counting up is like adding. (1.OA.C.5)
- Understand the meaning of the equal sign by determining if addition equations are true or false. (1.OA.D.7)
- Determine the unknown whole number in an addition equation relating three whole numbers. (1.OA.D.8)
- Add a two-digit number and a one-digit number (within 100). (1.NBT.C.4)
- Add a two-digit number and a multiple of 10 (within 100). (1.NBT.C.4)
- Solve addition equations (within 100) using concrete models or drawings. (1.NBT.C.4)
- Solve addition equations (within 100) using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. (1.NBT.C.4)
- Demonstrate understanding that adding two-digit numbers means adding the ones and then the tens. (1.NBT.C.4)
- Demonstrate understanding that when adding two-digit numbers, it is sometimes necessary to make a group of ten from the ones (regroup). (1.NBT.C.4)
- Add up to four two-digit numbers using strategies based on place value and properties of operations. (2.NBT.B.6)
- Use concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction to add within 1000. (2.NBT.B.7)
- Demonstrate understanding that adding three-digit numbers means adding hundreds and hundreds, tens and tens, ones and ones. (2.NBT.B.7)
- Demonstrate understanding that when adding three-digit numbers, it is sometimes necessary to compose or decompose tens or hundreds. (2.NBT.B.7)
- Explain why addition strategies work, using place value and the properties of operations. (2.NBT.B.9)

Problem Solving

- Use addition to solve word problems involving situations of adding to and putting together (within 20). (1.OA.A.1)
- Solve addition word problems, with unknowns in all positions. (1.OA.A.1)
- Use objects, drawings, and equations with a symbol for the unknown number to represent addition word problems. (1.OA.A.1; (1.OA.A.2))
- Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20. (1.OA.A.2)
- Use addition to solve one- and two-step word problems involving situations of adding to and putting together (within 100). (2.OA.A.1)
- Solve subtraction word problems, with unknowns in all positions. (2.OA.A.1)
- Use drawings and equations with a symbol for the unknown number to represent the problem. (2.OA.A.1)

Array

- Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns. (.2.OA.C.4)
- Write an equation to show the total number of objects are in an array. (.2.OA.C.4)

continues

ASSESSMENT CONSIDERATIONS

MIDDLE SERIES

Students will be asked to:

Problem Solving

- Solve two-step word problems that involve addition. (3.OA.D.8)
- Solve two-step word problems by writing an equation with a letter in place of the unknown number. (3.OA.D.8)
- Solve multi- step word problems involving whole numbers using knowledge of addition. (4.OA.A.3)
- Represent word problems by using equations with a letter standing for the unknown number. (4.OA.A.3)

Patterns

- Find patterns in addition tables and explain them using knowledge about how numbers work. (3.OA.D.9)
- Create a number or shape pattern that follows a given rule. (4.OA.C.5)
- Notice and point out different features of a pattern once it is created by a rule. (4.OA.C.5)

LATER SERIES

Students will be asked to:

Patterns

- Create two number patterns using two given rules. (5.OA.B.3)
- Identify relationships between two number patterns. (5.OA.B.3)

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)

OPERATIONS AND ALGEBRAIC THINKING (OA)

REPRESENT AND SOLVE PROBLEMS INVOLVING ADDITION AND SUBTRACTION

1.OA.A.1	Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
1.OA.A.2	Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
2.OA.A.1	Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

ADD AND SUBTRACT WITHIN 20

1.OA.C.5	Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).

continues

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)	
WORK WITH ADDITION AND SUBTRACTION EQUATIONS	
1.OA.D.7	Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.
1.OA.D.8	Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = _ - 3, 6 + 6 = _.
WORK WITH EQUAL GROUPS OF OBJECTS TO GAIN FOUNDATIONS FOR MULTIPLICATION	
2.OA.C.4	Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.
SOLVE PROBLEMS INVOLVING THE FOUR OPERATIONS, AND IDENTIFY AND EXPLAIN PATTERNS IN ARITHMETIC	
3.OA.D.8	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
3.OA.D.9	Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.
USE THE FOUR OPERATIONS WITH WHOLE NUMBERS TO SOLVE PROBLEMS	
4.OA.A.3	Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
GENERATE AND ANALYZE PATTERNS	
4.OA.C.5	Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

continues

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)

ANALYZE PATTERNS AND RELATIONSHIPS

5.OA.B.3	Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

NUMBER AND OPERATIONS IN BASE TEN

USE PLACE VALUE UNDERSTANDING AND PROPERTIES OF OPERATIONS TO ADD AND SUBTRACT

1.NBT.C.4	Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.
2.NBT.B.6	Add up to four two-digit numbers using strategies based on place value and properties of operations.
2.NBT.B.7	Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
2.NBT.B.9	Explain why addition and subtraction strategies work, using place value and the properties of operations.

NOTES

OPERATIONS: SUBTRACTION

SKILLS INVENTORY

Uses vocabulary for talking about addition (first addend, second addend, sum), subtraction (minuend, subtrahend, difference), multiplication (multiplicand, multiplier, product) and division (dividend, divisor, quotient).

Lower Elementary

Solves whole number mathematical equations and word-problems using all four operations with Montessori materials (to the materials full place value capacity) moving towards abstraction.

Upper Elementary

Solves whole number mathematical equations and word-problems using all four operations abstractly.

MONTESSORI LESSONS	PURPOSES
INITIAL SERIES	
OPERATIONS	
Golden Beads • Static • Dynamic	• To use Montessori materials, concrete models, and drawings to solve subtraction equations. • To solve subtraction equations using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. • To explain why subtraction strategies work, using place value and the properties of operations. • To give the sensorial impression of subtraction: Taking away smaller quantities from a larger quantity. • To understand that in subtracting multi-digit numbers, one subtracts ones and ones, tens and tens, hundreds and hundreds, etc. • To understand that in subtracting multi-digit numbers, sometimes it is necessary to compose, decompose a ten, hundred, thousand, etc.

continues

MONTESSORI LESSONS	PURPOSES
Stamp Game	• To use Montessori materials, concrete models, and drawings to solve subtraction equations. • To solve subtraction equations using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. • To explain why subtraction strategies work, using place value and the properties of operations. • To apply the symbols for the operations: + - x ÷ to the decimal system. • To understand the meaning of the equal sign, and determine if equations involving subtraction are true or false. • To understand that in subtracting multi-digit numbers, one subtracts ones and ones, tens and tens, hundreds and hundreds, etc. • To understand that in subtracting multi-digit numbers, sometimes it is necessary to compose, decompose a ten, hundred, thousand, etc.
Hierarchical Frames • Static • Dynamic	• To use Montessori materials, concrete models, and drawings to solve subtraction equations. • To solve subtraction equations using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. • To explain why subtraction strategies work, using place value and the properties of operations. • To provide the child with an opportunity to work in a more symbolic way with subtraction. • To offer an opportunity for the child to apply memorized knowledge of the essential combinations. • To reinforce that there are not more than 9 in any category in the decimal system. • To reinforce zero as a place holder. • To offer an opportunity to practice recording quantities. • To allow the child to eventually understand the algorithms for subtraction. • To understand that in subtracting multi-digit numbers, one subtracts ones and ones, tens and tens, hundreds and hundreds, etc. • To understand that in subtracting multi-digit numbers, sometimes it is necessary to compose, decompose a ten, hundred, thousand, etc. • To relate counting to subtraction.

continues

MONTESSORI LESSONS	PURPOSES
Unknown Number • Teacher-Created Lessons	• To determine the unknown whole number in a subtraction equation relating three whole numbers. • To understand subtraction as an unknown-addend problem.
Problem Solving • Teacher-Created or Purchased Cards • Experiences in the Classroom	• To verify the concepts of the operations of the decimal system. • To offer an opportunity to apply the knowledge of the operations to real-life situations. • Use subtraction to solve one- and two-step word problems involving situations of taking from, taking apart, and comparing.
MIDDLE SERIES	
Problem Solving • Teacher-Created or Purchased Cards • Experiences in the Classroom	• To solve multi-step word problems involving whole numbers using knowledge of subtraction. • To solve multi-step word problems by using equations with a letter standing for the unknown number.
Patterns • Teacher-Created Lessons	• To create a number or shape pattern that follows a given rule. • To notice and point out different features of a pattern once it is created by a rule.
LATER SERIES	
Patterns • Teacher-Created Lessons	• To create two number patterns using two given rules. • To identify relationships between two number patterns.

ASSESSMENT VOCABULARY

INITIAL SERIES

add
addition
compare
compose
compose a ten
concrete
count
decompose/ decomposition
equal
equation
false
hundreds
less than (<)
multiple of ten
ninety
one-digit number
ones
one-step problem
place value
properties of operations
put together
range (of numbers or data)
relationship
represent
strategy
subtract
subtraction
sum
symbol
take apart
take from
ten
tens
three
three-digit number
true
twenty
two
two-step problem
two-digit number
unknown
unknown – number problem
whole number
whole number
word problem

Cognitive Verbs

compare
compose
decompose
explain
relate
represent
solve
understand

MIDDLE SERIES

In addition to previous vocabulary:

addition table
estimation
four
mental math
multiplication table
multi step problem
operation
pattern
quantity
reasonable
remainder
rounding
rule

Cognitive Verbs

answer
assess
generate
identify
interpret
pose

LATER SERIES

In addition to previous vocabulary:

coordinate plane
corresponding terms
graph
ordered pair

Cognitive Verbs

form
graph

ASSESSMENT CONSIDERATIONS

INITIAL SERIES

Students will be asked to:

Operations

- Understand subtraction as an unknown-addend problem. (1.OA.B.4)
- Demonstrate understanding of how counting down is like subtracting. (1.OA.C.5)
- Demonstrate understanding of an equal sign by showing that a subtraction number sentence is true. (1.OA.D.7)
- Figure out what a missing number is in a subtraction problem. (1.OA.D.8)
- Use different strategies to subtract multiples of 10 (10-90) from numbers under 100, write the matching number sentence and explain their strategy. (1.NBT.C.6)
- Use strategies to subtract numbers within 1000 and know when to borrow. (2.NBT.B.7)
- Use knowledge of place value to explain why subtracting strategies work. (2.NBT.B.9)

Problem Solving

- Use different strategies for subtraction to solve word problems (within 20). (1.OA.A.1)
- Use strategies to solve subtraction word problems (within 100). (2.OA.A.1)

MIDDLE SERIES

Students will be asked to:

Problem Solving

- Solve two-step word problems that involve subtraction. (3.OA.D.8)
- Solve two-step word problems by writing an equation with a letter in place of the unknown number. (3.OA.D.8)
- Solve multi- step word problems involving whole numbers using knowledge of subtraction. (4.OA.A.3)
- Represent word problems by using equations with a letter standing for the unknown number. (4.OA.A.3)

Patterns

- Create a number or shape pattern that follows a given rule. (4.OA.C.5)
- Notice and point out different features of a pattern once it is created by a rule. (4.OA.C.5)

LATER SERIES

Students will be asked to:

Patterns

- Create two number patterns using two given rules. (5.OA.B.3)
- Identify relationships between two number patterns. (5.OA.B.3)

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)	
OPERATIONS AND ALGEBRAIC THINKING (OA)	
REPRESENT AND SOLVE PROBLEMS INVOLVING ADDITION AND SUBTRACTION	
1.OA.A.1	Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
1.OA.A.2	Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
2.OA.A.1	Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
UNDERSTAND AND APPLY PROPERTIES OF OPERATIONS AND THE RELATIONSHIP BETWEEN ADDITION AND SUBTRACTION	
1.OA.B.4	Understand subtraction as an unknown-addend problem. For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.
ADD AND SUBTRACT WITHIN 20	
1.OA.C.5	Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).
WORK WITH ADDITION AND SUBTRACTION EQUATIONS	
1.OA.D.7	Understand the meaning of the equal sign,and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.
1.OA.D.8	Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = _ - 3, 6 + 6 = _.
SOLVE PROBLEMS INVOLVING THE FOUR OPERATIONS, AND IDENTIFY AND EXPLAIN PATTERNS IN ARITHMETIC	
3.OA.D.8	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

continues

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)	
3.OA.D.9	Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.
USE THE FOUR OPERATIONS WITH WHOLE NUMBERS TO SOLVE PROBLEMS	
4.OA.A.3	Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
GENERATE AND ANALYZE PATTERNS	
4.OA.C.5	Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.
ANALYZE PATTERNS AND RELATIONSHIPS	
5.OA.B.3	Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.
NUMBER AND OPERATIONS IN BASE TEN (NBT)	
USE PLACE VALUE UNDERSTANDING AND PROPERTIES OF OPERATIONS TO ADD AND SUBTRACT	
1.NBT.C.4	Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.
1.NBT.C.6	Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

continues

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)

2.NBT.B.7	Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
2.NBT.B.9	Explain why addition and subtraction strategies work, using place value and the properties of operations.

NOTES

OPERATIONS: MULTIPLICATION

SKILLS INVENTORY

Lower Elementary

- Solves whole number mathematical equations and word-problems using all four operations with Montessori materials (to the materials full place value capacity) moving towards abstraction.
- Uses vocabulary for talking about addition (first addend, second addend, sum), subtraction (minuend, subtrahend, difference), multiplication (multiplicand, multiplier, product) and division (dividend, divisor, quotient).

Upper Elementary

- Solves whole number mathematical equations and word-problems using all four operations abstractly.
- Uses vocabulary for talking about addition (first addend, second addend, sum), subtraction (minuend, subtrahend, difference), multiplication (multiplicand, multiplier, product) and division (dividend, divisor, quotient).

MONTESSORI LESSONS	PURPOSES
INITIAL SERIES	
OPERATIONS	
Golden Beads	• To give the sensorial impression of multiplication: Putting together equal quantities to form a larger quantity. • To show that multiplication is the addition of equal quantities.
Stamp Game	• To apply the symbols for the operations: + - x ÷ to the decimal system. • To demonstrate that multiplication is just adding the same number a certain number of times.
MIDDLE SERIES	
OPERATIONS	
Stamp Game	• To apply the symbols for the operations: + - x ÷ to the decimal system. • To show the child that multiplication is just adding the same number a certain number of times. • To demonstrate understanding of multiplication by thinking about groups of objects. • To demonstrate understanding that multiplication equations can be seen as comparisons of groups. • To multiply a whole number up to four digits by a one-digit whole number. • To multiply two two-digit numbers. • To multiply multi-digit whole numbers using the standard algorithm.

continues

MONTESSORI LESSONS	PURPOSES
Hierarchical Frames • No Facts • With Facts	• To offer an opportunity for the child to apply memorized knowledge of the essential combinations. • To reinforce that there are not more than 9 in any category in the decimal system. • To reinforce zero as a place holder. • To offer an opportunity to practice recording quantities. • For children to move toward more abstract representations of multiplication and notation. • To emphasize the process of decomposing numbers into their component categories. • To multiply a whole number up to four digits by a one-digit whole number. • To multiply two two-digit numbers. • To multiply multi-digit whole numbers using the standard algorithm.

Checkerboard
• Reading Numbers
• One-digit Multiplier
• Multi-digit Multiplier
  • No Number Facts, No Writing
  • Some Facts, No Writing
  • Facts, Recording Problem and Final Product
  • Facts, Recording Partial Products
• Writing Products Directly
	• To provide a sensorial means to explore long multiplication of numbers up to hundred millions times thousands.
	• To provide an approach to multiplication leading naturally and logically to the standard multiplication algorithm.
	• For children to become familiar with partial products.
	• For children to notate multiplication problems, and partial and final products.
	• For the child to be introduced to the possibility that long multiplication problems can be solved using mental math.
	• To demonstrate understanding of multiplication by thinking about groups of objects.
	• To multiply any one digit whole number by a multiple of 10
	• To demonstrate understanding that multiplication equations can be seen as comparisons of groups.
	• To multiply a whole number up to four digits by a one-digit whole number.
	• To multiply two two-digit numbers.
	• To multiply multi-digit whole numbers using the standard algorithm.

continues

MONTESSORI LESSONS	PURPOSES
Flat Bead Frame • Multiplication by a two to four-digit Multiplier: Writing Final Product Only • Multiplication by a two to four-digit Multiplier: Writing Partial Products	• To multiply a whole number up to four digits by a one-digit whole number. • To multiply two two-digit numbers. • To multiply multi-digit whole numbers using the standard algorithm.
Bank Game • Multiplication by one-digit Multiplier • Multiplication by two-digit Multiplier • Multiplication by three-digit Number	• To demonstrate understanding of multiplication by thinking about groups of objects. • To multiply any one digit whole number by a multiple of 10 • To demonstrate understanding that multiplication equations can be seen as comparisons of groups. • To multiply a whole number up to four digits by a one-digit whole number. • To multiply two two-digit numbers. • To multiply multi-digit whole numbers using the standard algorithm.
Unknown Numbers • Teacher-Created Lessons	• To find the missing number in a multiplication equation. • To write an equation with a letter in place of the unknown number. • To multiply by understanding how multiplication and division are related.
Problem Solving • Teacher-Created or Purchased Cards • Experiences in the Classroom	• To verify the concepts of the operations of the decimal system. • To offer an opportunity to apply the knowledge of the operations to real-life situations. • To solve two-step word problems that involve multiplication. • To solve multiplication word problems by using drawings. • To solve multi- step word problems involving whole numbers using knowledge of multiplication. • To represent word problems by using equations with a letter standing for the unknown number.
Array • Teacher-Created Lessons	• To arrange objects in rows and columns to aid in multiplication. • To interpret a multiplication equation as a comparison. • To illustrate and explain how to multiply larger numbers by arrays.

continues

MONTESSORI LESSONS	PURPOSES
Model • Teacher-Created Lessons	• To use visuals or manipulatives to help interpret the numbers. • To illustrate and explain how to multiply larger numbers by using models.
Patterns • Teacher-Created Lessons	• To find patterns in multiplication tables and explain them using knowledge about how numbers work. • To create a number or shape pattern that follows a given rule. • To notice and point out different features of a pattern once it is created by a rule.
LATER SERIES	
Paper • Geometric Form of Multiplication • Cross Multiplication • Multiplication Algorithm	• To emphasize that multiplication can be separated into multiplying single-digit numbers and identifying categories. • To help the child explore an alternate approach to multiplying large numbers. • To realize that there is more than one way to produce a given decimal category. • To fluently multiply multi-digit whole numbers using the standard algorithm.
Patterns • Teacher-Created Lessons	• To find patterns of multiplication. • Identify arithmetic patterns and explain them using properties of operations. • Create two number patterns using two given rules. • Identify relationships between two number patterns.

ASSESSMENT VOCABULARY

MIDDLE SERIES

add
addition table
additive
array
comparison
divide
division
equal
equation
estimation
four
group/grouping
measurement
mental math
multi digit number
multiple of ten
multiplication
multiplication table
multiply
multi step problem
ninety
number
one-digit number
operation
pattern
place value
product
properties of operations
quantity
range (of numbers or data)
reasonable
relationship
remainder
represent
rounding
rule
standard algorithm
strategy
subtract
symbol
ten
three
total
two-step problem
unknown
whole number
word problem

Cognitive Verbs

answer
assess
determine
distinguish
explain
identify
interpret
pose
relate
represent
solve

LATER SERIES

In addition to previous vocabulary:
coordinate plane
corresponding terms
graph
ordered pair
two

Cognitive Verbs

form
generate
graph

ASSESSMENT CONSIDERATIONS

MIDDLE SERIES

Students will be asked to:

Operations

- Demonstrate understanding of multiplication by thinking about groups of objects. (3.OA.A.1)
- Find the missing number in a multiplication or division equation. (3.OA.A.4)
- Multiply within 100 easily and quickly by understanding how multiplication and division are related. (3.OA.C.7)
- Multiply any one-digit whole number by a multiple of 10 (6 x 90, 4 x 30). (3.NBT.A.3)
- Demonstrate understanding that multiplication equations can be seen as comparisons of groups. (4.OA.A.1)
- Multiply a whole number up to four digits by a one-digit whole number. (4.NBT.B.5)
- Multiply two two-digit numbers. (4.NBT.B.5)

Problem Solving

- Use knowledge about multiplication and division to solve word problems. (3.OA.A.3)
- Solve two-step word problems that involve multiplication. (3.OA.D.8)
- Solve two-step word problems by writing an equation with a letter in place of the unknown number. (3.OA.D.8)
- Multiply to solve word problems by using drawings or writing equations and solving for a missing number. (4.OA.A.2)
- Solve multi- step word problems involving whole numbers using knowledge of multiplication. (4.OA.A.3)
- Represent word problems by using equations with a letter standing for the unknown number. (4.OA.A.3)

Arrays

- Illustrate and explain how to multiply larger numbers by using equations, arrays or models. (4.NBT.B.5)

Models

- Illustrate and explain how to multiply larger numbers by using equations, arrays or models. (4.NBT.B.5)

Patterns

- Find patterns in multiplication tables and explain them using knowledge about how numbers work. (3.OA.D.9)
- Create a number or shape pattern that follows a given rule. (4.OA.C.5)
- Notice and point out different features of a pattern once it is created by a rule. (4.OA.C.5)

LATER SERIES

Students will be asked to:

Operations

- Fluently multiply multi-digit whole numbers using the standard algorithm. (5.NBT.B.5)

Patterns

- Create two number patterns using two given rules. (5.OA.B.3)
- Identify relationships between two number patterns. (5.OA.B.3)

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)

OPERATIONS AND ALGEBRAIC THINKING (OA)

REPRESENT AND SOLVE PROBLEMS INVOLVING MULTIPLICATION AND DIVISION	
3.OA.A.1	Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 × 7.
3.OA.A.3	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
3.OA.A.4	Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?
MULTIPLY AND DIVIDE WITHIN 100	
3.OA.C.7	Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.
SOLVE PROBLEMS INVOLVING THE FOUR OPERATIONS, AND IDENTIFY AND EXPLAIN PATTERNS IN ARITHMETIC	
3.OA.D.8	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
3.OA.D.9	Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.
USE THE FOUR OPERATIONS WITH WHOLE NUMBERS TO SOLVE PROBLEMS	
4.OA.A.1	Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
4.OA.A.2	Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.
4.OA.A.3	Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

continues

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)	
GENERATE AND ANALYZE PATTERNS	
4.OA.C.5	Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.
ANALYZE PATTERNS AND RELATIONSHIPS	
5.OA.B.3	Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.
NUMBER AND OPERATIONS IN BASE TEN (NBT)	
USE PLACE VALUE UNDERSTANDING AND PROPERTIES OF OPERATIONS TO PERFORM MULTI-DIGIT ARITHMETIC	
3.NBT.A.3	Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.
4.NBT.B.5	Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
PERFORM OPERATIONS WITH MULTI-DIGIT WHOLE NUMBERS AND WITH DECIMALS TO HUNDREDTHS	
5.NBT.B.5	Fluently multiply multi-digit whole numbers using the standard algorithm.

NOTES

OPERATIONS: DIVISION

SKILLS INVENTORY

Lower Elementary

- Solves whole number mathematical equations and word-problems using all four operations with Montessori materials (to the materials full place value capacity) moving towards abstraction.
- Uses vocabulary for talking about addition (first addend, second addend, sum), subtraction (minuend, subtrahend, difference), multiplication (multiplicand, multiplier, product) and division (dividend, divisor, quotient).

Upper Elementary

- Solves whole number mathematical equations and word-problems using all four operations abstractly.
- Uses vocabulary for talking about addition (first addend, second addend, sum), subtraction (minuend, subtrahend, difference), multiplication (multiplicand, multiplier, product) and division (dividend, divisor, quotient).

MONTESSORI LESSONS	PURPOSES
INITIAL SERIES	
OPERATIONS	
Golden Beads • Short Division • Long Division	• To give the sensorial impression of division: Sharing a quantity into a certain number of equal parts. • To show how quantities are distributed in Long Division. • To show that the answer in Division is the amount that one of the units in the divisor receives.
Stamp Game • Short Division • 2 Digit Divisor • 3-4 Digit Divisor • 0 in the Middle of the Divisor • 0 in the Units of the Divisor	• To apply the symbols for the operations: + - x ÷ , to the decimal system. • To explore group division.

continues

MONTESSORI LESSONS	PURPOSES
MIDDLE SERIES	
OPERATIONS	

Racks and Tubes
• One-digit Divisor
• Multi-digit Divisor
• Special Cases in Division
• Recording
  • Simple Recording
  • Record Problem, Quotient, Remainder
  • Record Intermediate Remainders, Quotient, Final Remainder
  • Record What has been used, Intermediate Remainders, Quotient, Final Remainder
	• To explore partitive division.
	• To demonstrate understanding of division by thinking about how one group can be divided into smaller groups.
	• To demonstrate understanding of how multiplication and division are related.
	• To find whole-number quotients and remainders.

Unknown Numbers • Teacher-Created Lessons	• To find the missing number in a multiplication or division equation. • To find the answer to a division problem by thinking of the missing factor in a multiplication problem. • To solve problems by writing an equation with a letter in place of the unknown number.
Problem Solving • Teacher-Created or Purchased Cards • Experiences in the Classroom	• To verify the concepts of the operations of the decimal system. • To offer an opportunity to apply the knowledge of the operations to real-life situations. • To use knowledge about multiplication and division to solve word problems. • To divide to solve word problems by using drawings or writing equations. • To solve multi- step word problems involving whole numbers using knowledge of division. • To represent word problems by using equations with a letter standing for the unknown number.
Array • Teacher-Created Lessons	• To arrange objects in rows and columns to aid in division. • To illustrate and explain how to divide larger numbers by using arrays.

continues

MONTESSORI LESSONS	PURPOSES
Model • Teacher-Created Lessons	• To use visuals or manipulatives to help interpret the numbers. • To illustrate and explain how to divide larger numbers by using models.
Patterns • Teacher-Created Lessons	• To find patterns of division. • To create patterns of division following a given rule. • To identify arithmetic patterns and explain them using properties of operations. • To create a number or shape pattern that follows a given rule. • To notice and point out different features of a pattern once it is created by a rule.
LATER SERIES	
OPERATIONS	
Paper • Division Algorithm	• To give the child a means of multiplying without any materials. • To fluently divide four-digit numbers (dividends) by two-digit numbers (divisors).
Array • Teacher-Created Lessons	• To arrange objects in rows and columns to aid in division. • To illustrate and explain a division problem using arrays.
Model • Teacher-Created Lessons	• To use visuals or manipulatives to help interpret the numbers. • To illustrate and explain a division problem using models.
Patterns • Teacher-Created Lessons	• To find patterns of division. • To create patterns of division following a given rule. • To create two number patterns using two given rules. • To identify relationships between two number patterns.

ASSESSMENT VOCABULARY

MIDDLE SERIES

addition table
additive
area model
array
calculation
divide
dividend
division
divisor
equal
equation
estimation
four
four-digit number
group/grouping
measurement
mental math
multiplication
multiplication table
multiply
multi step problem
number
one-digit number
operation
partition
pattern
place value
product
properties of operations
quantity
quotient
reasonable
rectangular array
relationship
remainder
represent
rounding
rule
share
strategy
symbol
three
two-step problem
unknown
unknown-factor problem
whole number
word problem

Cognitive Verbs

answer
assess
determine
distinguish
identify
interpret
partition
pose
relate
represent
solve
understand

LATER SERIES

In addition to previous vocabulary:

coordinate plane
corresponding terms
graph
ordered pair
two
two-digit number

Cognitive Verbs

explain
form
generate
graph
illustrate

ASSESSMENT CONSIDERATIONS

MIDDLE SERIES

Students will be asked to:

Operations

- Demonstrate understanding of division by thinking about how one group can be divided into smaller groups. (3.OA.A.2)
- Find the missing number in a multiplication or division equation. (3.OA.A.4)
- Find the answer to a division problem by thinking of the missing factor in a multiplication problem. (3.OA.B.6)
- Divide within 100 easily and quickly by understanding how multiplication and division are related. (3.OA.C.7)
- Find whole-number quotients and remainders with up to four-digit dividends and one- digit divisors. (4.NBT.B.6)

Problem Solving

- Use knowledge about multiplication and division to solve word problems. (3.OA.A.3)
- Solve two-step word problems that involve division. (3.OA.D.8)
- Solve two-step word problems by writing an equation with a letter in place of the unknown number. (3.OA.D.8)
- Divide to solve word problems by using drawings or writing equations and solving for a missing number. (4.OA.A.2)
- Solve multi- step word problems involving whole numbers using knowledge of division. (4.OA.A.3)
- Represent word problems by using equations with a letter standing for the unknown number. (4.OA.A.3)

Arrays

- Illustrate and explain how to divide larger numbers by using equations, arrays or models. (4.NBT.B.6)

Models

- Illustrate and explain how to divide larger numbers by using equations, arrays or models. (4.NBT.B.6)

Patterns

- Identify arithmetic patterns and explain them using properties of operations. (3.OA.D.9)
- Create a number or shape pattern that follows a given rule. (4.OA.C.5)
- Notice and point out different features of a pattern once it is created by a rule. (4.OA.C.5)

LATER SERIES

Students will be asked to:

Operations

- Divide four-digit numbers (dividends) by two-digit numbers (divisors). (5.NBT.B.6)

Arrays

- Illustrate and explain a division problem using equations, arrays and/or models. (5.NBT.B.6)

Models

- Illustrate and explain a division problem using equations, arrays and/or models. (5.NBT.B.6)

Patterns

- Create two number patterns using two given rules. (5.OA.B.3)
- Identify relationships between two number patterns. (5.OA.B.3)

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)

OPERATIONS AND ALGEBRAIC THINKING (OA)

REPRESENT AND SOLVE PROBLEMS INVOLVING MULTIPLICATION AND DIVISION

3.OA.A.2	Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.
3.OA.A.3	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
3.OA.A.4	Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?

UNDERSTAND PROPERTIES OF MULTIPLICATION AND THE RELATIONSHIP BETWEEN MULTIPLICATION AND DIVISION

3.OA.B.6	Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.

MULTIPLY AND DIVIDE WITHIN 100

3.OA.C.7	Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

SOLVE PROBLEMS INVOLVING THE FOUR OPERATIONS, AND IDENTIFY AND EXPLAIN PATTERNS IN ARITHMETIC

3.OA.D.8	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding
3.OA.D.9	Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

USE THE FOUR OPERATIONS WITH WHOLE NUMBERS TO SOLVE PROBLEMS

4.OA.A.2	Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

continues

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)	
4.OA.A.3	Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
GENERATE AND ANALYZE PATTERNS	
4.OA.C.5	Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.
ANALYZE PATTERNS AND RELATIONSHIPS	
5.OA.B.3	Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.
NUMBER AND OPERATIONS IN BASE TEN (NBT)	
USE PLACE VALUE UNDERSTANDING AND PROPERTIES OF OPERATIONS TO PERFORM MULTI-DIGIT ARITHMETIC	
4.NBT.B.6	Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
PERFORM OPERATIONS WITH MULTI-DIGIT WHOLE NUMBERS AND WITH DECIMALS TO HUNDREDTHS	
5.NBT.B.6	Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

NOTES

PROPERTIES OF NUMBERS

SKILLS INVENTORY

Lower Elementary

Applies properties of operations for addition (Associative & Commutative) and multiplication (Associative, Commutative & Distributive) to identify arithmetic patterns and as strategies to add, subtract, multiply and divide.

Upper Elementary

Applies the properties of addition (Associative & Commutative) and multiplication (Associative, Commutative & Distributive) to generate equivalent expressions.

MONTESSORI LESSONS	PURPOSES
INITIAL SERIES	
ADDITION: ASSOCIATIVE LAW	
Bead Bars • Single Parenthesis • Multiple Parentheses	• To introduce the associative law. • To provide a sensorial experience of the properties of operations. • To use addition facts to solve problems where there are more than two numbers (associative). • To explain arithmetic patterns using properties of operations.
ADDITION: COMMUTATIVE LAW	
Bead Bars	• To introduce the commutative law. • To provide a sensorial experience of the properties of operations. • To use fact families to solve addition problems (commutative).
Addition Strip Board and Charts	• To introduce the commutative law: the order of the addends does not affect the sum.
MIDDLE SERIES	
MULTIPLICATION: ASSOCIATIVE LAW	
Fluency	• To use the associative property of multiplication as a strategy for multiplying and dividing. • To explain arithmetic patterns using properties of operations.

continues

MONTESSORI LESSONS	PURPOSES
MULTIPLICATION: COMMUTATIVE LAW	
Bead Bars	• To use the commutative property of multiplication as a strategy for multiplying and dividing. • To explain arithmetic patterns using properties of operations.
Multiplication Strip Board and Charts	• To give a sensorial experience of the commutative law for multiplication. • For the children to understand the commutative law for multiplication: order isn't important for multiplication.
MULTIPLICATION: DISTRIBUTIVE LAW	
Bead Bars • Concept and Language • Sensorial Exploration with Signs • Introduction to Symbols • Symbolic Representation • Multiplication of Composite Numbers with Number Cards • Multiplication of Composite Numbers with Number Cards	• For children to understand the distributive law. • To use the distributive property of multiplication as a strategy for multiplying and dividing. • To explain arithmetic patterns using properties of operations.
LATER SERIES	
MULTIPLICATION: ASSOCIATIVE LAW	
Fluency	• To apply what is known about the associative law to create equivalent (or equal) expressions.
MULTIPLICATION: COMMUTATIVE LAW	
Fluency	• To apply what is known about the commutative law to create equivalent (or equal) expressions.
MULTIPLICATION: DISTRIBUTIVE LAW	
Fluency	• To use the distributive property to show the product of two whole numbers in different ways. • To apply what is known about the distributive law to create equivalent (or equal) expressions.

ASSESSMENT VOCABULARY

INITIAL SERIES	MIDDLE SERIES	LATER SERIES
add associative property commutative property properties of operations strategy subtract subtraction unknown unknown number problem **Cognitive Verbs** apply understand	*In addition to previous vocabulary:* addition table divide multiplication table multiply pattern **Cognitive Verbs** explain identify	*In addition to previous vocabulary:* common factor distributive property equal equivalent greatest common factor least common multiple less than (<) multiple one sum twelve two whole number **Cognitive Verbs** express generate

ASSESSMENT CONSIDERATIONS

INITIAL SERIES

ADDITION

Students will be asked to:

Associative

- Use addition facts to solve problems where there are more than two numbers (associative). Example: To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (1.OA.B.3)

Commutative

- Use fact families to solve addition problems (commutative). Example: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (1.OA.B.3)
- Understand subtraction as an unknown-addend problem (1.OA.B.4)

continues

ASSESSMENT CONSIDERATIONS

MIDDLE SERIES

MULTIPLICATION

Students will be asked to:

- Explain arithmetic patterns using properties of operations. (3.OA.D.9)

Associative

- Use the Associative property of multiplication as a strategy for multiplying and dividing. Example: $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (3.OA.B.5)

Commutative

- Use the commutative property of multiplication as a strategy for multiplying and dividing. Example: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (3.OA.B.5)

Distributive

- Use the distributive property of multiplication as a strategy for multiplying and dividing. Example: Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (3.OA.B.5)

LATER SERIES

MULTIPLICATION

Students will be asked to:

Distributive

- Use the distributive property to show the product of two whole numbers (1-100) in different ways. Example: express $36 + 8$ as $4 (9 + 2)$. (6.NS.B.4)

Equivalent Expressions

- Apply what is known about the properties of operations (associative, commutative and distributive) to create equivalent (or equal) expressions. (6.EE.A.3)

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)

OPERATIONS AND ALGEBRAIC THINKING (OA)

UNDERSTAND AND APPLY PROPERTIES OF OPERATIONS AND THE RELATIONSHIP BETWEEN ADDITION AND SUBTRACTION.

1.OA.B.3	Apply properties of operations as strategies to add and subtract. Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)

UNDERSTAND PROPERTIES OF MULTIPLICATION AND THE RELATIONSHIP BETWEEN MULTIPLICATION AND DIVISION.

3.OA.B.5	Apply properties of operations as strategies to multiply and divide. Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)

SOLVE PROBLEMS INVOLVING THE FOUR OPERATIONS, AND IDENTIFY AND EXPLAIN PATTERNS IN ARITHMETIC.

3.OA.D.9	Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

THE NUMBER SYSTEM (NS)

COMPUTE FLUENTLY WITH MULTI-DIGIT NUMBERS AND FIND COMMON FACTORS AND MULTIPLES.

6.NS.B.4	Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the Distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2).

EXPRESSIONS AND EQUATIONS (EE)

APPLY AND EXTEND PREVIOUS UNDERSTANDINGS OF ARITHMETIC TO ALGEBRAIC EXPRESSIONS.

6.EE.A.3	Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.

NOTES

MEMORIZATION

SKILLS INVENTORY

Lower Elementary

Memorizes addition (0-10), subtraction (0-10), multiplication (0-12) and division (0-12) facts.

Upper Elementary

Demonstrates automaticity, speed and accuracy of number fact recall and application to mastery level.

MONTESSORI LESSONS	PURPOSES
INITIAL SERIES	
ADDITION	
Addition Snake Game • Counting the Snake • Control of Error: Matching the 10s • Two by Two	• To give the first practice in memorization of the essential combinations in addition. • To help the child experience the fact that no two quantities less than ten can make more than eighteen. • To reenforce all of the possible combinations that make ten. • To use strategies such as counting on, making ten, decomposing a number leading to a ten. • To create equivalent but easier or known sums.
Addition Strip Board • Using the Tables • Exploration • How Many Ways? • Cut Combinations	• To guide the child through all of the possible essential combinations of addition in a systematic fashion. • To show that no essential combination exists beyond 9 + 9 = 18. • To reinforce the mechanism of addition helping the child to see and memorize the essential combinations of addition.
Addition Chart • Whole Board • Half Board • Simplified Board • Blank Board • Chart of Sums	• To mentally find 10 more than the number. • To fluently add within 100 using strategies based on the relationship between addition and subtraction.

continues

MONTESSORI LESSONS	PURPOSES
Fluency	• To know all sums of two one-digit numbers from memory by end of grade 2. • To demonstrate fluency for addition within 20. • To mentally add 10 more to a given two-digit number. • To mentally add 10 or 100 to a given number 100-900.
SUBTRACTION	
Subtraction Snake Game • Counting the Snake • Control of Error • Two by Two	• To familiarize the child with all of the essential combinations in subtraction. • To mentally find 10 less than the number, without having to count and explain the reasoning used.
Subtraction Strip Board • Using the Tables • Exploration • How Many Ways? • Cut Combinations	• To practice subtraction, leading to the memorization of the essential combinations in subtraction. • To show that subtraction is the opposite action of addition.
Subtraction Chart • Whole Finger Board • Blank Board • Chart of Differences	• To use the relationship between addition and subtraction. • To fluently subtract within 100 using strategies based on the relationship between addition and subtraction.
Fluency	• To mentally find 10 less than a given two-digit number. • To mentally subtract 10 or 100 from a given number 100-900. • To fluently subtract within 20 using mental strategies. • To fluently subtract within 100 using strategies.
MIDDLE SERIES	
ADDITION	
Fluency	• To fluently add within 1,000 using strategies and algorithms. • To fluently add multi-digit whole numbers.
SUBTRACTION	
Fluency	• To fluently subtract within 1,000 using strategies and algorithms. • To fluently subtract multi-digit whole numbers.

continues

MONTESSORI LESSONS	PURPOSES
MULTIPLICATION	
Multiplication Bead Bars • Exploring the Tables • How Many Ways? • Multiplying by 10	• To familiarize the child with all of the essential combination in multiplication.
Multiplication Bead Board	• To practice multiplication leading to memorization of the essential combinations.
Multiplication Chart • Whole Board • Half Board • Simplified Board • Blank Board	• To see the relationship between multiplication and division. • To divide easily because of knowledge of how multiplication and division are related. • To easily divide multi-digit numbers.
Fluency	• To fluently multiply within 100. • To know all products of two one-digit numbers from memory by the end of Grade 3.
DIVISION	
Unit Division Board • Using the Board • Division from 81	• To show that not every quantity is evenly divisible, and to show that some quantities are divisible only by a few numbers. • To discover the essential combinations of division. • To practice division leading to the memorization of the essential combinations of division.
Division Chart • Whole Finger Board • Blank Board • Chart of Quotients	• To see the relationship between multiplication and division. • To divide within 100 easily because of knowledge of how multiplication and division are related. • To easily divide multi-digit numbers.
Fluency	• To fluently divide within 100.
LATER SERIES	
DIVISION	
Fluency	• To easily divide multi-digit numbers. • To divide within 100 easily because of knowledge of how multiplication and division are related.

ASSESSMENT VOCABULARY

INITIAL SERIES	MIDDLE SERIES	LATER SERIES
add/ addition count count on decompose/decomposition equivalent less make ten mental math more number one-digit number place value properties of operations relationship strategy subtract/ subtraction sum two-digit number **Cognitive Verbs** create decompose demonstrate explain	*In addition to previous vocabulary:* add/ addition algorithm divide/ division multi digit number multiplication/ multiply one-digit number place value product properties of operations relationship standard algorithm strategy subtract/ subtraction whole number	*In addition to previous vocabulary:* divide multi digit number standard algorithm

ASSESSMENT CONSIDERATIONS

INITIAL SERIES

ADDITION

Students will be asked to:

- Add within 20. (1.OA.C.6)
- Demonstrate fluency for addition within 10. (1.OA.C.6)
- Use strategies such as counting on, making ten, decomposing a number leading to a ten. (1.OA.C.6)
- Use the relationship between addition and subtraction. (1.OA.C.6)
- Create equivalent but easier or known sums. (1.OA.C.6)
- Given a two-digit number, mentally find 10 more than the number, without having to count and explain the reasoning used. (1.NBT.C.5)
- Fluently add within 20 using mental strategies. (2.OA.B.2)
- Know from memory all sums of two one-digit numbers from memory by end of grade 2. (2.OA.B.2)
- Fluently add within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. (2.NBT.B.5)
- Mentally add 10 or 100 to a given number 100-900. (2.NBT.B.8)

SUBTRACTION

Students will be asked to:

- Subtract within 20. (1.OA.C.6)
- Use the relationship between addition and subtraction. (1.OA.C.6)
- Given a two-digit number, mentally find 10 less than the number, without having to count and explain the reasoning used. (1.NBT.C.5)
- Fluently subtract within 20 using mental strategies. (2.OA.B.2)
- Fluently subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. (2.NBT.B.5)
- Mentally subtract 10 or 100 from a given number 100-900. (2.NBT.B.8)

MIDDLE SERIES

ADDITION

Students will be asked to:

- Fluently add within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. (3.NBT.A.2)
- Fluently add multi-digit whole numbers. (4.NBT.B.4)

continues

ASSESSMENT CONSIDERATIONS

SUBTRACTION

Students will be asked to:

- Fluently subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. (3.NBT.A.2)
- Fluently subtract multi-digit whole numbers. (4.NBT.B.4)

MULTIPLICATION

Students will be asked to:

- Fluently multiply within 100, using strategies such as the relationship between multiplication and division or properties of operations. (3.OA.C.7)
- By the end of Grade 3, know from memory all products of two one-digit numbers. (3.OA.C.7)

DIVISION

Students will be asked to:

- Fluently divide within 100, using strategies such as the relationship between multiplication and division or properties of operations. (3.OA.C.7)

LATER SERIES

DIVISION

Students will be asked to:

- Easily divide multi-digit numbers. (6.NS.B.2)
- Divide within 100 easily because of knowledge of how multiplication and division are related. (6.NS.B.2)

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)

OPERATIONS AND ALGEBRAIC THINKING (OA)

ADD AND SUBTRACT WITHIN 20

1.OA.C.6	Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).
2.OA.B.2	Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.

continues

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)	
MULTIPLY AND DIVIDE WITHIN 100	
3.OA.C.7	Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.
NUMBER AND OPERATIONS IN BASE TEN (NBT)	
USE PLACE VALUE UNDERSTANDING AND PROPERTIES OF OPERATIONS TO ADD AND SUBTRACT	
1.NBT.C.5	Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.
2.NBT.B.5	Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
2.NBT.B.8	Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100-900.
USE PLACE VALUE UNDERSTANDING AND PROPERTIES OF OPERATIONS TO PERFORM MULTI-DIGIT ARITHMETIC	
3.NBT.A.2	Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
4.NBT.B.4	Fluently add and subtract multi-digit whole numbers using the standard algorithm.
THE NUMBER SYSTEM	
COMPUTE FLUENTLY WITH MULTI-DIGIT NUMBERS AND FIND COMMON FACTORS AND MULTIPLES	
6.NS.B.2	Fluently divide multi-digit numbers using the standard algorithm.

NOTES

MULTIPLES, FACTORS, AND DIVISIBILITY

SKILLS INVENTORY

Upper Elementary

Demonstrates understanding of multiples, factors and divisibility, including common multiple, least common multiple, common factor, greatest common factor, composite and prime numbers.

MONTESSORI LESSONS	PURPOSES
INITIAL SERIES	
MULTIPLES	

Bead Chains and Bead Bars
• Concept and Language with Short Bead Chains
• Concept and Language with the Bead Bars
  • Multiples of Single-digit Numbers
  • Multiples of Multi-digit Numbers
• Concept and Language of the Common Multiple
	• For children to understand that the multiples of a number are groups of that number multiple times.
	• To introduce the concept and terms of multiples.
	• To practice multiplication facts and skip counting.
	• To introduce the concept and term of "common multiple".
	• To provide the child with physical representation of the concept of common multiples in preparation for work with fractions and other advanced mathematical practices.

MIDDLE SERIES	
MULTIPLES	
Pegboard • Least Common Multiples (LCM) • Least Common Multiples with Hierarchical Colors	• To introduce the concept and term of "least common multiple" and abbreviation LCM. • To recognize a whole number as a multiple of each of its factors. • To determine whether a whole number is a multiple of a given one-digit number. • To determine whether a given whole number is a prime or composite number.
Paper Charts • Multiples of Numbers Paper • Tables A & B • Table C: Concept and Language for Prime Numbers • Sieve of Eratosthenes	• To investigate all multiples from 2-100. • To practice multiplication facts and skip counting.

continues

MONTESSORI LESSONS	PURPOSES
FACTORS	
• Pegboard • Concept of Factors • Common Factors • Greatest Common Factors	• To introduce the concept and language "factor". • To reinforce the geometry of multiplication. • To introduce the concept and term of "common factor". • To introduce the concept and term of "greatest common factors" and abbreviation "GCF". • To find all factor pairs for a whole number.
Paper • Finding Prime Factors on Paper • Finding Least Common Multiples using Prime Factorization • Finding Greatest Common Factors using Prime Factorization	• To introduce the concept and term of "prime factor". • To introduce a method for calculating prime factors. • To set children on the path of concluding that the LCM is the product of the union of the prime factors of all the numbers. • To give the child tools for finding factors without materials. • To give the child an important tool for simplifying work with fractions and arithmetic.
DIVISIBILITY	
Golden Bead Material • Divisibility by 2, 5, 25 • Divisibility by 4, 8 • Chart for Divisibility • Divisibility by 3, 6, and 9 • Divisibility by 11 • Divisibility by 7	• To introduce the concept and term of "divisibility". • To learn the rules for divisibility. • To simplify and reinforce work in factoring, multiples, divisibility, and fractions. • To further develop the child's number sense through a knowledge of divisibility. • For the child to use a chart in order to track the divisibility of the same number by different quantities.
LATER SERIES	
MULTIPLES	
Paper	• To find the least common multiple of two whole numbers.
FACTORS	
Paper	• To mentally compare the size of a product to the size of one of the factors by thinking about the other factor in the problem. • To find the greatest common factor of two whole numbers.
DIVISIBILITY	
Paper • Divisibility by Prime Factors	• For the child to ascertain divisibility in a more abstract fashion, utilizing prior knowledge.

ASSESSMENT VOCABULARY

MIDDLE SERIES	LATER SERIES
composite number factor factor pair multiple one range (of numbers or data) whole number **Cognitive Verbs** determine recognize	*In addition to previous vocabulary:* common factor distributive property equal greatest common factor least common multiple less than (<) sum twelve two **Cognitive Verbs** express

ASSESSMENT CONSIDERATIONS

MIDDLE SERIES

MULTIPLES

Students will be asked to:

- Recognize a whole number as a multiple of each of its factors. (4.OA.B.4)
- Determine whether a whole number from 1 to 100 is a multiple of a given one-digit number. (4.OA.B.4)
- Determine whether a given whole number up to 100 is a prime or composite number. (4.OA.B.4)

FACTORS

Students will be asked to:

- Find all factor pairs for a whole number from 1 to 100. (4.OA.B.4)

LATER SERIES

MULTIPLES

Students will be asked to:

- Find the least common multiple of two whole numbers less than or equal to 12. (6.NS.B.4)

FACTORS

Students will be asked to:

- Mentally compare the size of a product to the size of one of the factors by thinking about the other factor in the problem. (5.NF.B.5.A)
- Find the greatest common factor of two whole numbers less than or equal to 100. (6.NS.B.4)

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)

OPERATIONS AND ALGEBRAIC THINKING (OA)

GAIN FAMILIARITY WITH FACTORS AND MULTIPLES

4.OA.B.4	Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.

NUMBER AND OPERATIONS: FRACTIONS (NF)

APPLY AND EXTEND PREVIOUS UNDERSTANDINGS OF MULTIPLICATION AND DIVISION

5.NF.B.5.A	Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

THE NUMBER SYSTEM (NS)

COMPUTE FLUENTLY WITH MULTI-DIGIT NUMBERS AND FIND COMMON FACTORS AND MULTIPLES

6.NS.B.4	Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2).

NOTES

CHAPTER 5

FRACTIONS

FOUNDATIONS

SKILLS INVENTORY

Lower Elementary

Partition shapes (circle, rectangle) into equal shares and express these areas as fractions (written and spoken). Order fractions on a number line and compare fractions.

Upper Elementary

Compare two fractions by demonstrating understanding that fractions can only be compared when they refer to the same whole (have the same denominator)

MONTESSORI LESSONS	PURPOSES
INITIAL SERIES	
Introduction to Fractions • Impressionistic Introduction • Concept of Notation	• To introduce the concept of fractions with a sensorial experience.
Common Fractions • Quantities and Names • Symbols	• To introduce the concept of fractions with a sensorial experience. • To partition circles into equal shares. • To describe the shares of partitioned circles using words (halves, thirds, fourths, etc.). • To describe the shares of partitioned circles using phrases (half of, fourth of, quarter of, etc.). • To describe the whole of partitioned circles as two halves, three thirds, four fourths, etc.
Fractions as Part of a Set	• To understand that fractions can also be used to describe parts of a set as opposed to sections of a divided whole. • To understand that a group of individual items can also be discussed in terms of fractions.

continues

MONTESSORI LESSONS	PURPOSES

Equivalent Figure Material
• Other Representations for Fractions
  • Metal Square: nine plates
  • Metal Triangles: four plates
	• To introduce the idea that a unit can be any shape or size.
	• To partition a rectangle into rows and columns of equal squares.
	• To partition rectangles into equal shares.
	• To describe the shares of partitioned rectangles using words (halves, thirds, fourths, etc.).
	• To describe the shares of partitioned rectangles using phrases (half of, fourth of, quarter of, etc.).
	• To describe the whole of partitioned rectangles as two halves, three thirds, four fourths, etc.

MIDDLE SERIES	
Common Fractions • Quantities, Names & Symbols • Mixed Numbers & Fractions Greater than one	• To express the area of each part as a unit fraction of the whole. • To understand a fraction (1/b) as the quantity formed by 1 part when a whole is partitioned into b equal parts. • To understand a fraction (a/b) as the quantity formed by a parts of size 1/b. • To express whole numbers as fractions. • To understand and express mixed numbers.
Number Lines • Teacher-Created Lessons	• To place Montessori fraction circles on a number line. • To label fractions on a number line. • To understand that the space between two numbers on the number line are a whole. • To show a fraction on a number line by marking off equal parts between two whole numbers.
Comparing Fractions • Teacher-Created Lessons	• To compare fractions with the symbols >, =, <. • To use Montessori fractions circles to prove comparisons. • To compare fractions by reasoning about their size. • To compare two fractions with the same numerator or the same denominator. • To recognize that comparisons are valid only when the two fractions refer to the same whole. • To compare two fractions with different numerators and different denominators.

ASSESSMENT VOCABULARY

INITIAL SERIES

circle
column
count
decompose/decomposition
equal
fourth (fraction)
fourth (fraction)
half
identical
more
number
partition
quarter (one-fourth)
rectangle
row
shape
share
small/smaller
square
third (fraction)
total
whole

Cognitive Verbs

create
decompose
describe
partition
recognize
understand

MIDDLE SERIES

In addition to previous vocabulary:

area
benchmark fraction
common denominator
common numerator
compare
comparison
denominator
endpoint
equal
equivalent
fraction
interval
length
number
number line
numerator
part
partition
quantity
represent
shape
size
symbol
unit fraction
valid
visual fraction model
whole
whole number
zero

Cognitive Verbs

compare
define
express
form
justify
locate
reason
record
refer
represent

ASSESSMENT CONSIDERATIONS

INITIAL SERIES

Students will be asked to:

- Partition circles into two and four equal shares. (1.G.A.3)
- Partition rectangles into two and four equal shares. (1.G.A.3)
- Describe the shares of partitioned circles and rectangles using the words halves, fourths, and quarters. (1.G.A.3)
- Use the phrases half of, fourth of, and quarter of to describe the shares of partitioned circles and rectangles. (1.G.A.3)
- Describe the whole as two of, or four of the shares of the partitioned circles and rectangles. (1.G.A.3)
- Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. (2.G.A.2)
- Partition circles into two, three, or four equal shares. (2.G.A.3)
- Partition rectangles into two, three, or four equal shares. (2.G.A.3)
- Describe the shares of partitioned circles and rectangles using the words halves, thirds, half of, a third of, etc. (2.G.A.3)
- Describe the whole of partitioned circles and rectangles as two halves, three thirds, four fourths. (2.G.A.3)

MIDDLE SERIES

Students will be asked to:

- Express the area of each part as a unit fraction of the whole. (3.G.A.2)
- Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts. (3.NF.A.1)
- Understand a fraction a/b as the quantity formed by a parts of size 1/b. (3.NF.A.1)
- Understand a fraction as a number on the number line. (3.NF.A.2)
- Represent fractions on a number line diagram. (3.NF.A.2)
- Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. (3.NF.A.2.A)
- Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line. (3.NF.A.2.A)
- Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. (3.NF.A.2.B)
- Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line. (3.NF.A.2.B)
- Compare fractions by reasoning about their size. (3.NF.A.3)
- Express whole numbers as fractions. (3.NF.A.3.C)
- Compare two fractions with the same numerator or the same denominator by reasoning about their size. (3.NF.A.3.D)
- Recognize that comparisons of 2 fractions are valid only when the two fractions refer to the same whole. (3.NF.A.3.D)
- Record the results of comparisons with the symbols >, =, or <, and justify the conclusions. (3.NF.A.3.D)
- Compare two fractions with different numerators and different denominators. (4.NF.A.2)
- Recognize that comparisons are valid only when the two fractions refer to the same whole. (4.NF.A.2)
- Record the results of comparisons with symbols >, =, or <, and justify the conclusions. (4.NF.A.2)

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)	
GEOMETRY (G)	
REASON WITH SHAPES AND THEIR ATTRIBUTES	
1.G.A.3	Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.
2.G.A.2	Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
2.G.A.3	Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.
3.G.A.2	Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as ¼ of the area of the shape.
NUMBER AND OPERATIONS: FRACTIONS (NF)	
DEVELOP UNDERSTANDING OF FRACTIONS AS NUMBERS	
3.NF.A.1	Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.
3.NF.A.2	Understand a fraction as a number on the number line; represent fractions on a number line diagram.
3.NF.A.2.A	Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.
3.NF.A.2.B	Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.
3.NF.A.3	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
3.NF.A.3.C	Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.

continues

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)	
3.NF.A.3.D	Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
EXTEND UNDERSTANDING OF FRACTION EQUIVALENCE AND ORDERING	
4.NF.A.2	Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as ½. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

NOTES

EQUIVALENCE

SKILLS INVENTORY

Lower Elementary

Understand and show that fractions of different sizes and shapes can be equivalent if they have the same value or the same point on a number line.

Upper Elementary

Recognize and generate equivalent fractions by demonstrating how the number and size of parts can differ in fractions with the same value.

MONTESSORI LESSONS	PURPOSES
INITIAL SERIES	
Fraction Circles • Equivalency of Fractions	• To introduce the concept of fractions with a sensorial experience. • To understand that decomposing fraction pieces into more equal shares creates smaller shares.
Equivalent Figure Material • Metal Square: nine plates • Metal Triangles: four plates	• To recognize that equal shares of identical wholes need not have the same shape.
MIDDLE SERIES	
Fraction Circles • Equivalency of Fractions	• To introduce the concept of fractions with a sensorial experience. • To partition circles into parts with equal areas. • To understand two fractions as equivalent if they are the same size. • To recognize and generate simple equivalent fractions. • To explain why 2 fractions are equivalent. • To recognize fractions that are equivalent to whole numbers.
Equivalent Figure Material • Metal Square: 9 plates • Metal Triangles: 4 plates	• To explain why 2 fractions are equivalent. • To partition shapes into parts with equal areas. • To recognize and generate simple equivalent fractions. • To recognize fractions that are equivalent to whole numbers. • To understand that equivalent fractions can be represented by different shapes. • To understand two fractions as equivalent if they are the same size.

continues

MONTESSORI LESSONS	PURPOSES
Paper	• To express a fraction with denominator 10 as an equivalent fraction with denominator 100.
Number Lines • Teacher-Created Lessons	• To place equivalent Montessori fraction circles on a number line. • To label equivalent fractions on a number line. • To understand two equivalent fractions, including whole numbers, are the same point on a number line.

ASSESSMENT VOCABULARY

INITIAL SERIES	MIDDLE SERIES
circle decompose/decomposition equal fourth (fraction) half identical more partition quarter (one-fourth) rectangle shape share small/smaller third (fraction) whole **Cognitive Verbs** create decompose describe partition recognize understand	*In addition to previous vocabulary:* add area denominator equivalent fraction number number line part point simple fraction size unit fraction visual fraction model whole number **Cognitive Verbs** explain express generate

ASSESSMENT CONSIDERATIONS

INITIAL SERIES

Students will be asked to:

- Understand that decomposing fraction pieces into more equal shares creates smaller shares. (1.G.A.3)
- Recognize that equal shares of identical wholes need not have the same shape. (2.G.A.3)

MIDDLE SERIES

Students will be asked to:

- Partition shapes into parts with equal areas. (3.G.A.2)
- Explain equivalence of fractions in special cases. (3.NF.A.3)
- Understand two fractions as equivalent (equal) if they are the same size. (3.NF.A.3.A)
- Understand two equivalent fractions, including whole numbers, are the same point on a number line. (3.NF.A.3.A; 3.NF.A.3.C)
- Recognize and generate simple equivalent fractions. (3.NF.A.3.B)
- Explain why the fractions are equivalent. (3.NF.A.3.B)
- Recognize fractions that are equivalent to whole numbers. (3.NF.A.3.C)
- Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models. (4.NF.A.1)
- Explain how the number and size of fractional parts differ even though the two fractions themselves are the same size. (4.NF.A.1)
- Recognize and generate equivalent fractions by using visual fraction models. (4.NF.A.1)
- Express a fraction with denominator 10 as an equivalent fraction with denominator 100. (4.NF.C.5)

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)

GEOMETRY (G)

REASON WITH SHAPES AND THEIR ATTRIBUTES

1.G.A.3	Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.
2.G.A.3	Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.
3.G.A.2	Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.

continues

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)

NUMBER AND OPERATIONS: FRACTIONS (NF)

DEVELOP UNDERSTANDING OF FRACTIONS AS NUMBERS

3.NF.A.3	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
3.NF.A.3.A	Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
3.NF.A.3.B	Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
3.NF.A.3.C	Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.

EXTEND UNDERSTANDING OF FRACTION EQUIVALENCE AND ORDERING

4.NF.A.1	Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

UNDERSTAND DECIMAL NOTATION FOR FRACTIONS, AND COMPARE DECIMAL FRACTIONS

4.NF.C.5	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.2 For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.

NOTES

ADDITION AND SUBTRACTION

SKILLS INVENTORY

Lower Elementary

Add and subtract fractions sensorially using Montessori materials.

Upper Elementary

Add and subtract fractions with unlike denominators (including mixed numbers) and solve addition and subtraction of fraction word problems using knowledge of fractions and number sense to estimate and assess the reasonableness of sums and differences.

MONTESSORI LESSONS	PURPOSES
INITIAL SERIES	

Fraction Circles
• Operations, Simple Cases
  • Addition: Same Denominators
  • Subtraction: Same Denominators
	• To form knowledge through experience joining fraction pieces together.
	• To provide a sensorial experience of adding fractions.
	• To form knowledge through experience separating fractions pieces.
	• To provide a sensorial experience of subtracting fractions.

MIDDLE SERIES	
Fraction Circles • Operations beyond Simple Cases • Addition and Subtraction: Different Denominators, Sensorial	• To provide a sensorial experience of adding and subtracting fractions with different denominators. • To provide a sensorial experience of finding common denominators. • To understand addition fractions as joining parts referring to the same whole. • To understand subtraction of fractions as separating parts referring to the same whole.
Mixed Numbers • Addition • Subtraction	• To add and subtract mixed numbers with like denominators by replacing each mixed number with an equivalent fraction. • To add and subtract mixed numbers with like denominators by using properties of operations. • To add mixed numbers with like denominators by using the relationship between addition and subtraction.

continues

MONTESSORI LESSONS	PURPOSES
Transparencies • Operations beyond Simple Cases • Addition and Subtraction: Alternate Methods for Finding Common Denominators	• To provide a sensorial experience of adding and subtracting fractions with different denominators. • To provide a sensorial experience of finding common denominators. • To decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. • To justify the decompositions of a fraction.
Problem Solving • Teacher-Created or Purchased Cards • Experiences in the Classroom	• For children to solve addition and subtraction word problems. • To solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators. • To use visual fraction models and equations to represent addition and subtraction word problems.
LATER SERIES	

Paper
• Operations beyond Simple Cases
  • Addition and Subtraction: Alternate Methods for Finding Common Denominators using Graph Paper
  • Addition and Subtraction: Finding and Using the LCD
  • Addition and Subtraction: Passage to Abstraction
	• For children to move to adding and subtracting fractions without using the fraction materials.
	• To add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum of fractions with like denominators.
	• To use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.

Problem Solving • Teacher-Created or Purchased Cards • Experiences in the Classroom	• For children to solve addition and subtraction word problems. • To solve word problems involving adding and subtracting fractions referring to the same whole, including cases of unlike denominators. • To use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.

ASSESSMENT VOCABULARY

MIDDLE SERIES	LATER SERIES
add addition decompose/decomposition denominator equation equivalent fraction mixed number part properties of operations relationship represent subtract subtraction sum ten two visual fraction model whole word problem **Cognitive Verbs** decompose express justify record refer replace represent solve understand	*In addition to previous vocabulary:* benchmark fraction difference estimation mental math reasonable unlike denominators word problem **Cognitive Verbs** answer assess estimate produce

ASSESSMENT CONSIDERATIONS

MIDDLE SERIES

Students will be asked to:

- Understand a fraction a/b with a > 1 as a sum of fractions 1/b. (4.NF.B.3)
- Understand addition fractions as joining parts referring to the same whole. (4.NF.B.3.A)
- Understand subtraction of fractions as separating parts referring to the same whole. (4.NF.B.3.A)
- Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. (4.NF.B.3.B)
- Justify the decompositions of a fraction. (4.NF.B.3.B)
- Add mixed numbers with like denominators by replacing each mixed number with an equivalent fraction. (4.NF.B.3.C)
- Add mixed numbers with like denominators by using properties of operations. (4.NF.B.3.C)
- Add mixed numbers with like denominators by using the relationship between addition and subtraction. (4.NF.B.3.C)
- Subtract mixed numbers with like denominators by replacing each mixed number with an equivalent fraction. (4.NF.B.3.C)
- Subtract mixed numbers with like denominators by using properties of operations. (4.NF.B.3.C)
- Solve word problems involving addition of fractions referring to the same whole and having like denominators. (4.NF.B.3.D)
- Use visual fraction models and equations to represent addition word problems. (4.NF.B.3.D)
- Solve word problems involving subtraction of fractions referring to the same whole and having like denominators. (4.NF.B.3.D)
- Use visual fraction models and equations to represent subtraction word problem. (4.NF.B.3.D)
- Use knowledge of equivalent fractions to add two fractions with respective denominators 10 and 100. (4.NF.C.5)

LATER SERIES

Students will be asked to:

- Add fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum of fractions with like denominators. (5.NF.A.1)
- Subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent difference of fractions with like denominators. (5.NF.A.1)
- Solve word problems involving addition of fractions referring to the same whole, including cases of unlike denominators. (5.NF.A.2)
- Solve word problems involving subtraction of fractions referring to the same whole, including cases of unlike denominators. (5.NF.A.2)
- Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. (5.NF.A.2)

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)	
NUMBER AND OPERATIONS: FRACTIONS (NF)	
BUILD FRACTIONS FROM UNIT FRACTIONS	
4.NF.B.3	Understand a fraction a/b with a > 1 as a sum of fractions 1/b.
4.NF.B.3.A	Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
4.NF.B.3.B	Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.
4.NF.B.3.C	Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
4.NF.B.3.D	Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
UNDERSTAND DECIMAL NOTATION FOR FRACTIONS, AND COMPARE DECIMAL FRACTIONS	
4.NF.C.5	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.2 For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.
USE EQUIVALENT FRACTIONS AS A STRATEGY TO ADD AND SUBTRACT FRACTIONS	
5.NF.A.1	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)
5.NF.A.2	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.

NOTES

MULTIPLICATION

SKILLS INVENTORY

Lower Elementary

Multiply fractions sensorially using Montessori materials.

Upper Elementary

Multiply fractions (fraction by whole number, fraction by fraction, mixed numbers) and solve multiplication fraction word problems using knowledge of fractions and number sense to estimate and assess the reasonableness of the product.

MONTESSORI LESSONS	PURPOSES
INITIAL SERIES	
Fraction Circles • Operations, Simple Cases • Multiplication by a Whole Number	• To provide a sensorial experience of multiplying fractions.
MIDDLE SERIES	
Fraction Circles • Operations beyond Simple Cases • Multiplication by Whole Number: Sensorial	• To apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
Problem Solving • Teacher-Created or Purchased Cards • Experiences in the Classroom	• To solve word problems involving multiplication of a fraction by a whole number.

continues

MONTESSORI LESSONS	PURPOSES
LATER SERIES	
Fraction Circles • Operations beyond Simple Cases • Multiplication by Fraction: Sensorial	• To apply and extend previous understandings of multiplication to multiply a fraction by a fraction. • To explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number. • To explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number.
Paper • Operations beyond Simple Cases • Multiplication by Fractions using Graph Paper • Multiplication by Fractions: Passage to Abstraction	• For the children to become familiar with the algorithm for multiplying fractions.
Problem Solving • Teacher-Created or Purchased Cards • Experiences in the Classroom	• To solve real world problems involving multiplication of fractions and mixed numbers.

ASSESSMENT VOCABULARY	
MIDDLE SERIES	**LATER SERIES**
equation fraction multiple multiplication multiply represent visual fraction model whole number word problem **Cognitive Verbs** represent solve understand	*In addition to previous vocabulary:* mixed number real-world problem resize **Cognitive Verbs** apply interpret

ASSESSMENT CONSIDERATIONS

MIDDLE SERIES

Students will be asked to:

- Apply and extend previous understandings of multiplication to multiply a fraction by a whole number (4.NF.B.4; 5.NF.B.4)
- Understand a fraction a/b as a multiple of 1/b. (4.NF.B.4.A)
- Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. (4.NF.B.4.B)
- Solve word problems involving multiplication of a fraction by a whole number. (4.NF.B.4.C)

LATER SERIES

Students will be asked to:

- Apply and extend previous understandings of multiplication to multiply a fraction by a fraction. (5.NF.B.4)
- Interpret the product (a/b) × q as parts of a partition of q into b equal parts equivalently, as the result of a sequence of operations a × q ÷ b. (5.NF.B.4.A)
- Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. (5.NF.B.4.B)
- Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. (5.NF.B.4.B)
- Interpret multiplication as scaling (resizing). (5.NF.B.5)
- Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case). (5.NF.B.5.B)
- Explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number. (5.NF.B.5.B)
- Relate the principle of fraction equivalence a/b = (n × a)/(n × b) to the effect of multiplying a/b by 1. (5.NF.B.5.B)
- Solve real world problems involving multiplication of fractions and mixed numbers. (5.NF.B.6)

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)

NUMBER AND OPERATIONS: FRACTIONS (NF)

BUILD FRACTIONS FROM UNIT FRACTIONS

4.NF.B.4	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
4.NF.B.4.A	Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).
4.NF.B.4.B	Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)

continues

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)	
4.NF.B.4.C	Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?
APPLY AND EXTEND PREVIOUS UNDERSTANDINGS OF MULTIPLICATION AND DIVISION	
5.NF.B.4	Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
5.NF.B.4.A	Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = (ac)/(bd).
5.NF.B.4.B	Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
5.NF.B.5	Interpret multiplication as scaling (resizing), by:
5.NF.B.5.B	Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence a/b = (n × a)/(n × b) to the effect of multiplying a/b by 1.
5.NF.B.6	Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

NOTES

DIVISION

SKILLS INVENTORY

Lower Elementary

Divide fractions sensorially using Montessori materials.

Upper Elementary

Divide fractions (fraction by whole number, fraction by fraction, mixed numbers) and solve division fraction word problems using knowledge of fractions and number sense to estimate and assess the reasonableness of the quotient.

MONTESSORI LESSONS	PURPOSES
INITIAL SERIES	
Fraction Circles • Operations, Simple Cases • Division by a Whole Number	• To provide a sensorial experience of dividing fractions.
MIDDLE SERIES	

Fraction Circles
• Operations beyond Simple Cases
  • Division by a Whole Number: Partitive/sharing, Sensorial
  • Division by a Fraction: Partitive/sharing, Sensorial
	• For children to understand that the quotient of proper fractions is larger than the dividend.
	• To bring to the child's awareness that the quotient is given by the amount that the whole unit receives.

LATER SERIES	

Fraction Circles
• Operations beyond Simple Cases
  • Division by a Whole Number
  • Division by a Fraction
	• To interpret a fraction as division of the numerator by the denominator.
	• To apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.
	• To interpret division of a unit fraction by a non-zero whole number, and compute such quotients.
	• To interpret division of a whole number by a unit fraction, and compute such quotients.

Paper • Operations beyond Simple Cases • Division by Fractions: Passage to Abstraction	• For the children to become familiar with the algorithm for dividing fractions.

continues

MONTESSORI LESSONS	PURPOSES
Problem Solving • Teacher-Created or Purchased Cards • Experiences in the Classroom	• To solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions. • To solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers.

ASSESSMENT VOCABULARY

LATER SERIES

denominator	quotient	**Cognitive Verbs**
division	real-world problem	answer
equation	represent	compute
fraction	unit fraction	form
mixed number	visual fraction model	interpret
non-zero	whole number	represent
numerator	word problem	solve

ASSESSMENT CONSIDERATIONS

LATER SERIES

Students will be asked to:

- Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). (5.NF.B.3)
- Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers. (5.NF.B.3)
- Interpret the product (a/b) × q as parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. (5.NF.B.4.A)
- Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (5.NF.B.7)
- Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. (5.NF.B.7.A)
- Interpret division of a whole number by a unit fraction, and compute such quotients. (5.NF.B.7.B)
- Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions. (5.NF.B.7.C)
- Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions. (6.NS.A.1)

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)

NUMBER AND OPERATIONS: FRACTIONS (NF)

APPLY AND EXTEND PREVIOUS UNDERSTANDINGS OF MULTIPLICATION AND DIVISION

5.NF.B.3	Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?
5.NF.B.4.A	Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = (ac)/(bd).
5.NF.B.7	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.
5.NF.B.7.A	Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.
5.NF.B.7.B	Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.
5.NF.B.7.C	Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, "How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally?", "How many 1/3-cup servings are in 2 cups of raisins?".

THE NUMBER SYSTEM (NS)

APPLY AND EXTEND PREVIOUS UNDERSTANDINGS OF MULTIPLICATION AND DIVISION TO DIVIDE FRACTIONS BY FRACTIONS

6.NS.A.1	Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) "How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally?", "How many 3/4-cup servings are in 2/3 of a cup of yogurt?", "How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?".

NOTES

CHAPTER 6

DECIMALS

FOUNDATIONS

SKILLS INVENTORY
Upper Elementary Read, write, and compare decimals recognizing that a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

MONTESSORI LESSONS	PURPOSES
MIDDLE SERIES	
Decimal Board • Introduction to Decimal Fractions • Introduction to Quantity and Language • Introduction to Symbolic Notation for Decimals	• For the children to become familiar with the categories from tenths to millionths. • To give experience of the language for the place value of decimals. • To use decimal notation for fractions with denominators 10 or 100.
Conversion of Common Fractions to and from Decimal Fractions • Teacher-Created Lesson	• To rewrite a decimal as a fraction.
Number Lines • Teacher-Created Lesson	• To locate a decimal on a number line diagram.
Comparing Fractions • Teacher-Created Lesson	• To compare two decimals to hundredths by reasoning about their size. • To recognize that comparisons are valid only when the two decimals refer to the same whole. • To record the results of comparisons with the symbols >, =, or <, and justify the conclusions.
LATER SERIES	
Decimal Board • Formation in Cards and Reading of Multi-digit Decimals	• To read and write decimals. • To read and write decimals using base-ten numerals. • To read and write decimals using number names. • To read and write decimals in expanded form.

continued

MONTESSORI LESSONS	PURPOSES
Comparing Fractions • Teacher-Created Lesson	• To compare decimals. • To recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right. • To recognize that in a multi-digit number, a digit in one place represents 1/10 of what it represents in the place to its left. • To compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
Rounding • Teacher-Created Lesson	• To use place value understanding to round decimals to any place. • To build an understanding of "rounding up" and "rounding down" at different places in a decimal number.

ASSESSMENT VOCABULARY

MIDDLE SERIES	LATER SERIES
compare comparison decimal decimal notation denominator fraction hundredths size symbol ten two valid visual model whole **Cognitive Verbs** compare justify reason recognize record refer	*In addition to previous vocabulary:* base-ten numeral digit expanded form multi digit number number name one place place value represent rounding tenths thousandths two **Cognitive Verbs** represent

ASSESSMENT CONSIDERATIONS

MIDDLE SERIES

Students will be asked to:

- Use decimal notation for fractions with denominators 10 or 100. (4.NF.C.6)
- Rewrite a decimal as a fraction (0.62 as 62/100.) (4.NF.C.6)
- Locate a decimal (0.62) on a number line diagram. (4.NF.C.6)
- Compare two decimals to hundredths by reasoning about their size. (4.NF.C.7)
- Recognize that comparisons are valid only when the two decimals refer to the same whole. (4.NF.C.7)
- Record the results of comparisons with the symbols >, =, or <, and justify the conclusions. (4.NF.C.7)

LATER SERIES

Students will be asked to:

- Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right. (5.NBT.A.1)
- Recognize that in a multi-digit number, a digit in one place represents 1/10 of what it represents in the place to its left. (5.NBT.A.1)
- Read decimals to thousandths. (5.NBT.A.3)
- Write decimals to thousandths. (5.NBT.A.3)
- Compare decimals to thousandths. (5.NBT.A.3)
- Read decimals to thousandths using base-ten numerals. (5.NBT.A.3.A)
- Read decimals to thousandths using number names. (5.NBT.A.3.A)
- Read decimals to thousandths in expanded form. (5.NBT.A.3.A)
- Write decimals to thousandths using base-ten numerals. (5.NBT.A.3.A)
- Write decimals to thousandths using number names. (5.NBT.A.3.A)
- Write decimals to thousandths using expanded form. (5.NBT.A.3.A)
- Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. (5.NBT.A.3.B)
- Use place value understanding to round decimals to any place. (5.NBT.A.4)

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)

NUMBER AND OPERATIONS: FRACTIONS (NF)

UNDERSTAND DECIMAL NOTATION FOR FRACTIONS, AND COMPARE DECIMAL FRACTIONS

4.NF.C.6	Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.
4.NF.C.7	Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

NUMBER AND OPERATIONS IN BASE TEN (NBT)

UNDERSTAND THE PLACE VALUE SYSTEM

5.NBT.A.1	Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
5.NBT.A.3	Read, write, and compare decimals to thousandths.
5.NBT.A.3.A	Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$.
5.NBT.A.3.B	Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
5.NBT.A.4	Use place value understanding to round decimals to any place.

NOTES

OPERATIONS

SKILLS INVENTORY

Upper Elementary

Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

MONTESSORI LESSONS	PURPOSES
MIDDLE SERIES	
Decimal Board: Operations • Addition Using Decimal Board • Subtraction Using Decimal Board • Multiplication by Unit Multiplier using Decimal Board • Multiplication using Multi-digit Quantities on the Decimal Board • Division by Unit Divisor	• To provide a sensorial experience of working with decimal fractions.

Decimal Checkerboard
• Introduction to Decimal Checkerboard
• Multiplying a Decimal by a Whole Number
• Multiplication By a Decimal Fraction
	• To give the child a visual impression of the place value results of whole number and decimal multiplication.
	• To give the child the experience with multiplication of whole numbers, mixed numbers, and decimals.
	• For the child to perform these calculations:
	  • Recording problem and final answer only.
	  • Recording problem, partial products for final answer.

Skittles • Division with Decimals, beyond Simple Cases	• To provide a sensorial experience of dividing decimal fractions.
LATER SERIES	
Decimal Board: Operations • Addition Using Decimal Board • Subtraction Using Decimal Board • Multiplication by Unit Multiplier using Decimal Board • Multiplication using Multi-digit Quantities on the Decimal Board • Division by Unit Divisor	• To add, subtract, multiply, and divide decimals using concrete models or drawings, strategies based on place value, and strategies based on properties of operations.

continues

MONTESSORI LESSONS	PURPOSES
Decimal Checkerboard • Decimal Felt Squares	• To explain patterns in the number of zeros of the product when multiplying a number by powers of 10. • To explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10.
Skittles • Dividing a Decimal Fraction by a Unit Divisor • Division with Decimals, beyond Simple Cases	• To divide decimals using concrete models or drawings, strategies based on place value, and strategies based on properties of operations.
Paper • Addition of Decimal Fractions: Passage to Abstraction • Subtraction of Decimal Fractions: Passage to Abstraction • Multiplication with Paper Only • Division of Decimals on Paper Only	• To relate the strategy of adding decimals to a written method and explain the reasoning used. • To fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm. • To add decimals using strategies based on the relationship between addition and subtraction. • To relate the strategies of subtracting, multiplying, and dividing decimals to written methods and explain the reasoning used. • To understand that multiplying or dividing by a power of ten merely requires moving the decimal point.

ASSESSMENT VOCABULARY

LATER SERIES

add
addition
concrete
decimal
decimal point
divide
exponent
hundredths
multiply
number
pattern
place value
power of 10
product
properties of operations
relationship
strategy
subtract
subtraction
whole number
zero

Cognitive Verbs
explain
relate

ASSESSMENT CONSIDERATIONS

LATER SERIES

PATTERNS

Students will be asked to:

- Explain patterns in the number of zeros of the product when multiplying a number by powers of 10. (5.NBT.A.2)
- Explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. (5.NBT.A.2)

ADDITION

Students will be asked to:

- Add decimals to hundredths, using concrete models or drawings. (5.NBT.B.7)
- Add decimals to hundredths, using strategies based on place value. (5.NBT.B.7)
- Add decimals to hundredths, using strategies based on properties of operations. (5.NBT.B.7)
- Add decimals to hundredths, using strategies based on the relationship between addition and subtraction. (5.NBT.B.7)
- Relate the strategy of adding decimals to hundredths to a written method and explain the reasoning used. (5.NBT.B.7)
- Fluently add multi-digit decimals using the standard algorithm. (6.NS.B.3)

SUBTRACTION

Students will be asked to:

- Subtract decimals to hundredths, using concrete models or drawings. (5.NBT.B.7)
- Subtract decimals to hundredths, using strategies based on place value. (5.NBT.B.7)
- Subtract decimals to hundredths, using strategies based on properties of operations. (5.NBT.B.7)
- Subtract decimals to hundredths, using strategies based on the relationship between addition and subtraction. (5.NBT.B.7)
- Relate the strategy of subtracting decimals to hundredths to a written method and explain the reasoning used. (5.NBT.B.7)
- Fluently subtract multi-digit decimals using the standard algorithm. (6.NS.B.3)

MULTIPLICATION

Students will be asked to:

- Multiply decimals to hundredths, using concrete models or drawings. (5.NBT.B.7)
- Multiply decimals to hundredths, using strategies based on place value. (5.NBT.B.7)
- Multiply decimals to hundredths, using strategies based on properties of operations. (5.NBT.B.7)
- Relate the strategy of multiplying decimals to hundredths to a written method and explain the reasoning used. (5.NBT.B.7)
- Fluently multiply multi-digit decimals using the standard algorithm. (6.NS.B.3)

continues

ASSESSMENT CONSIDERATIONS

DIVISION

Students will be asked to:

- Divide decimals to hundredths, using concrete models or drawings. (5.NBT.B.7)
- Divide decimals to hundredths, using strategies based on place value. (5.NBT.B.7)
- Divide decimals to hundredths, using strategies based on properties of operations. (5.NBT.B.7)
- Relate the strategy of dividing decimals to hundredths to a written method and explain the reasoning used. (5.NBT.B.7)
- Fluently divide multi-digit decimals using the standard algorithm. (6.NS.B.3)

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)

NUMBER AND OPERATIONS IN BASE TEN (NBT)

UNDERSTAND THE PLACE VALUE SYSTEM

5.NBT.A.2	Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

PERFORM OPERATIONS WITH MULTI-DIGIT WHOLE NUMBERS AND WITH DECIMALS TO HUNDREDTHS

5.NBT.B.7	Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

THE NUMBER SYSTEM (NS)

COMPUTE FLUENTLY WITH MULTI-DIGIT NUMBERS AND FIND COMMON FACTORS AND MULTIPLES

6.NS.B.3	Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

NOTES

CHAPTER 7

ALGEBRA

POWERS OF NUMBERS

SKILLS INVENTORY

Lower Elementary

Demonstrates understanding of the concept and notation for squares and cubes of numbers using Montessori materials.

Upper Elementary

Demonstrates understanding of the concept and notation for squaring, cubing, square root, cube root and understands the order of operations when using exponents.

MONTESSORI LESSONS	PURPOSES
INITIAL SERIES	
SQUARES AND CUBES OF NUMBERS	
Bead Cabinet (Short Chains, Long Chains, Bead Squares and Cubes) • Exploration of Square Chains to Make Polygons • Exploration of Cube Chains to Make Polygons • Concept and Notation of a Square • Concept and Notation of Cube • Exploration of Cubes of Numbers 1-10 • Tower of Jewels	• To introduce the concept of the square of a number. • To observe that a square is both a geometric and an arithmetic concept. • To learn the symbolic notation for the square of a number. • To introduce the concept of the cube of a number. • To observe that a cube is both a geometric and an arithmetic concept. • To learn the symbolic notation for the cube of a number.

continues

MONTESSORI LESSONS	PURPOSES
MIDDLE SERIES	
SQUARING	
Paper Decanomial	• To create a paper model of the geometric form of squaring.
Bead Material • Building a Larger Square from a Smaller Square • Squaring a Sum (One-digit Terms) Application to Decimal Numbers (two Digits, Products to 999)	• To consolidate recognition of the geometric pattern of a binomial square. • To prepare for the square root and for calculating binomial squares abstractly.
Golden Beads • Transformation of Squares • Squaring a Binomial using Golden Beads • Application to Decimal Numbers using Golden Beads	• To recognize that the square of a binomial (the sum of two terms) creates a specific pattern of two squares along the diagonal and two rectangles filling in the gaps. • To identify similar patterns for the squares of other polynomials. • To introduce language for polynomials. • To recognize that we can apply the concept of polynomials to the decimal system.
LATER SERIES	
Exponential Notation • Operations with Exponential Notation • Special case: Multiplication of Powers of Numbers Having the Same Base • Special case: Division of Powers of Numbers Having the Same Base	• To introduce alternative methods for calculation using the four operations. • For children to learn the multiplication with powers of numbers (same base).

continues

MONTESSORI LESSONS	PURPOSES
SQUARING	
Pegboard • From the Real Square to the Symbolic Square • Squaring a Binomial using the Pegboard • Squaring a Trinomial using the Pegboard • Squaring Polynomials Using Hierarchical Pegs and Guide Squares	• To square single digit to multi-digit numbers.

Paper
• Algebraic Passages: Formula for Squaring a Binomial
• Algebraic Passages: Formula for Squaring a Trinomial
• Algebraic Passages: Formula for Squaring Polynomials
• Transformation of Squares
  • Paper Squares
  • Graph Paper
	• For the child to find the square on paper.

CUBING	
Cubing Material • From a Given Cube to a Successive Cube • From a Given Cube to a Non-successive Cube	• To provide a sensorial image of the composition of a binomial cube.
Binomial Cube • Cubing a Binomial Starting from the Square • Cubing a Binomial: Algebraic	• To extend the child's understanding of the meaning of cubing (third power) to a binomial using the distributive law. • To become familiar with the concept of an algebraic formula as a pattern, and the idea of plugging numbers into a formula to get a result.
Trinomial Cube • Cubing a Trinomial: Algebraic	• To become familiar with the concept of an algebraic formula as a pattern, and the idea of plugging numbers into a formula to get a result.

continues

MONTESSORI LESSONS	PURPOSES

Hierarchical Trinomial Cube
• Application to the Decimal System
  • The Story of the Three Rulers and Introducing the Hierarchical Trinomial
  • Cubing a Decimal Number using the Hierarchical Cube
	• For children to work with cubing larger numbers.
	• To provide opportunities to physically experience abstract concepts.

SQUARE ROOT	
Bead Material • Concept, Language, Notation of Square Root • Golden Bead Material • Golden Beads for Numbers under 1000	• To understand that the square root of a number "n" is another number "b" whose second power equals "n", (b x b = n or b2 = n). • To understand that geometrically, the square root can be read from the value of the appropriate edge of a square.
Square Root Board • Extracting a Square Root for Numbers ≤225 (Square Root Board and Loose Units)	• To have a sensorial experience of finding a square root of a number under 100.
Pegboard • Pegboard for Calculating Square Roots • Building the Square by Category • Building the Square by Periods • Calculating the Next Root Digit	• For the children to recognize that the number of digits in a number determine the number of digits in its root. • To challenge the child's powers of concentration and reasoning. • To move a step closer to calculating the square root without materials by completing the square one level at a time and avoiding backtracking. • For the children to recognize that the number of digits in a number determine the number of digits in its root.
Paper • Calculating the Square Root on Paper	• To arrive at an algorithm which does not rely upon material.

continues

MONTESSORI LESSONS	PURPOSES
CUBE ROOT	
Cubing Material and White 2 cm cubes • Concept, Language, Notation for Cube Root • Finding the Cube Root: four–six Digits with Wooden Cubing Material • Finding Cube Root: seven–nine digits using Hierarchical Trinomial Cube • Finding Cube Root Abstractly	• To provide children with the language to express abstract concepts. • For children to move to calculating cube root without using the materials.

ASSESSMENT VOCABULARY

Students will not be assessed on *Powers of Numbers*.

ASSESSMENT CONSIDERATIONS

Students will not be assessed on *Powers of Numbers*.

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)

Standards for Exponents are found beginning in 8th grade.

NOTES

SIGNED NUMBERS

SKILLS INVENTORY

Upper Elementary

Understand that positive and negative numbers are used together to describe quantities having opposite directions or values, explain the meaning of 0, position positive and negative numbers on a number line and complete operations with positive and negative numbers.

MONTESSORI LESSONS	PURPOSES
LATER SERIES	
Negative Snake Game • Addition of Signed Numbers • Subtraction of Signed Numbers • Multiplication of Signed Numbers • Division of Signed Numbers • Division with Grouping	• To understand that the full system of integers (positive and negative numbers together) is crucial for future work in algebra and graphing. • To give a sensorial experience of the behavior of signed numbers: positive and negative numbers "cancel" each other. • To understand that positive and negative numbers are used together to describe quantities having opposite directions or values. • To recognize that the opposite of the opposite of a number is the number itself. • To recognize that 0 is its own opposite.
Number Line and Coordinate Plane • Teacher-Created Lessons	• To recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line. • To find and position integers and other rational numbers on a horizontal or vertical number line diagram. • To find and position pairs of integers and other rational numbers on a coordinate plane. • To interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.

Absolute Value
• Teacher-Created Lessons
  • Number Line
  • Ordering
	• To understand the absolute value of a rational number as its distance from 0 on the number line.
	• To understand ordering and absolute value of rational numbers.
	• To distinguish comparisons of absolute value from statements about order.

continues

MONTESSORI LESSONS	PURPOSES
Paper • Addition • Subtraction • Multiplication • Division	• To perform math equations with positive and negative numbers without materials.
Problem Solving • Teacher-Created or Purchased Cards • Experiences in the Classroom	• To write, interpret, and explain statements of order for rational numbers in real-world contexts. • To use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. • To interpret absolute value as magnitude for a positive or negative quantity in a real-world situations.

ASSESSMENT VOCABULARY

LATER SERIES

absolute value
comparison
coordinate plane
diagram
direction
distance
horizontal number line
integer
magnitude
negative
number
number line
pair
positive
quantity
rational number
real-world context
real-world problem
represent
side
signed number
statement of inequality
statement of order
temperature
value
vertical number line diagram
zero

Cognitive Verbs
describe
distinguish
explain
interpret
recognize
represent
understand

ASSESSMENT CONSIDERATIONS

LATER SERIES

Students will be asked to:

- Understand that positive and negative numbers are used together to describe quantities having opposite directions or values. (6.NS.C.5)
- Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line. (6.NS.C.6.A)
- Recognize that the opposite of the opposite of a number is the number itself. (6.NS.C.6.A)
- Recognize that 0 is its own opposite. (6.NS.C.6.A)
- Find and position integers and other rational numbers on a horizontal or vertical number line diagram. (6.NS.C.6.C)
- Find and position pairs of integers and other rational numbers on a coordinate plane. (6.NS.C.6.C)
- Understand ordering and absolute value of rational numbers. (6.NS.C.7)
- Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. (6.NS.C.7.A)
- Write, interpret, and explain statements of order for rational numbers in real-world contexts. (6.NS.C.7.B)
- Understand the absolute value of a rational number as its distance from 0 on the number line. (6.NS.C.7.C)
- Distinguish comparisons of absolute value from statements about order. (6.NS.C.7.D)

Word Problems

- Use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. (6.NS.C.5)
- Interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. (6.NS.C.7.C)

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)	
THE NUMBER SYSTEM (NS)	
APPLY AND EXTEND PREVIOUS UNDERSTANDINGS OF NUMBERS TO THE SYSTEM OF RATIONAL NUMBERS	
6.NS.C.5	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
6.NS.C.6.A	Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3, and that 0 is its own opposite.
6.NS.C.6.C	Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
6.NS.C.7	Understand ordering and absolute value of rational numbers.
6.NS.C.7.A	Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret -3 > -7 as a statement that -3 is located to the right of -7 on a number line oriented from left to right.
6.NS.C.7.B	Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, "Write -3o C > -70 C to express the fact that -3o C is warmer than -7o C".
6.NS.C.7.C	Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of -30 dollars, write |-30| = 30 to describe the size of the debt in dollars.
6.NS.C.7.D	Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.

NOTES

Rational numbers are numbers that can be expressed as a fraction or part of a whole number.

Irrational numbers are numbers that cannot be expressed as a fraction or ratio of two integers.

Absolute value is a term used in mathematics to indicate the distance of a point or number from the origin (zero point) of a number line or coordinate system.

ALGEBRA

SKILLS INVENTORY

Upper Elementary

Write, read, and evaluate numerical expressions in which letters stand for numbers, perform arithmetic operations, applying the properties of operations to generate equivalent expressions.

MONTESSORI LESSONS	PURPOSES
LATER SERIES	
BEAD BARS	
Parentheses/Brackets/Braces	• To use parentheses, brackets, or braces to calculate numerical expressions.
Concept of Balancing an Equation	• To recognize that one can perform operations on a balanced equation without changing the balance, as long as the operation is performed on both sides. • To develop comfort with thinking of the equals sign as meaning "these two things are the same" instead of "the result of the left side is the right side". • To write simple expressions that record calculations with numbers. • To view one or more parts of an expression as a single entity. • To identify when two expressions are equivalent.
Solving for One Unknown	• To understand the concept of solving for a variable. • To introduce The Laws of Inverse operations. • To develop facility solving algebraic equations in one unknown. • To write, read, and calculate expressions in which letters stand for numbers. • To understand that a variable can represent an unknown number or any number in a specified set.

continues

MONTESSORI LESSONS	PURPOSES
Translating Verbal Problems into Equations	• For the children to competently translate numerical and real life verbal problems into equations. • To consolidate knowledge of the four operations and recognize them in verbal form. • To interpret numerical expressions without calculating them. • To write expressions that record operations with numbers and with letters standing for numbers. • To identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient).
Solving for Two Unknowns	• To write, read, and calculate expressions in which letters stand for numbers. • To understand that a variable can represent an unknown number or any number in a specified set.
Graphs of Algebraic Expressions	• To experience the connection between algebraic expressions and graphing. • To represent solutions of inequalities on number line diagrams.
Order of Operations	• To write and calculate numerical expressions involving whole-number exponents. • To perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). • To apply the properties of operations to generate equivalent expressions.
Inequalities	• To understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? • To use substitution to determine whether a given number in a specified set makes an equation or inequality true. • To recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions.
Dependent and Independent Variables	• To write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. • To analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

continues

MONTESSORI LESSONS	PURPOSES
Problem Solving • Teacher-Created or Purchased Cards • Experiences in the Classroom	• To offer an opportunity to apply the knowledge of algebra to real-life situations. • To prepare the child to tackle real-life mathematical problems. • To calculate expressions that arise from formulas used in real-world problems. • To use variables to represent numbers and write expressions when solving a real-world or mathematical problem. • To solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. • To write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. • To use variables to represent two quantities in a real-world problem that change in relationship to one another.

ASSESSMENT VOCABULARY

LATER SERIES

braces
brackets
calculation
coefficient
constraint
dependent variable
equation
equivalent
exponent
expression
factor
formula
graph
independent variable
inequality
infinitely many
mathematical problem
number
number line
numerical expression
one
operation
order of operations
parentheses
part
product
properties of operations
quantity
quotient
rational number
real-world problem
relationship
represent
set
simple expression
solution
sum
surface area
symbol
table
term
true
two
unknown
value
variable
whole number

Cognitive Verbs
analyze
answer
apply
determine
evaluate
express
form
generate
graph
identify
interpret
name
recognize
record
relate
represent
solve
substitute
understand

ASSESSMENT CONSIDERATIONS

LATER SERIES

Students will be asked to:

- Use parentheses, brackets, or braces in numerical expressions. (5.OA.A.1)
- Calculate expressions with parentheses, brackets, or braces. (5.OA.A.1)
- Write simple expressions that record calculations with numbers, and interpret numerical expressions without calculating them. (5.OA.A.2)
- Write and calculate numerical expressions involving whole-number exponents. (6.EE.A.1)
- Write, read, and calculate expressions in which letters stand for numbers. (6.EE.A.2)
- Write expressions that record operations with numbers and with letters standing for numbers. (6.EE.A.2.A)
- Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient). (6.EE.A.2.B)
- View one or more parts of an expression as a single entity. (6.EE.A.2.B)
- View (8 + 7) as both a single entity and a sum of two terms. (6.EE.A.2.B)
- Calculate expressions at specific values of their variables. (6.EE.A.2.C)
- Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). (6.EE.A.2.C)
- Apply the properties of operations to generate equivalent expressions; apply properties of operations to y + y + y to produce the equivalent expression 3y. (6.EE.A.3)
- Identify when two expressions are equivalent. (6.EE.A.4)
- Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? (6.EE.B.5)
- Use substitution to determine whether a given number in a specified set makes an equation or inequality true. (6.EE.B.5)
- Understand that a variable can represent an unknown number or any number in a specified set. (6.EE.B.6)
- Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions. (6.EE.B.8)
- Represent solutions inequalities on number line diagrams. (6.EE.B.8)
- Write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. (6.EE.C.9)
- Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. (6.EE.C.9)

Word Problems

- Calculate expressions that arise from formulas used in real-world problems. (6.EE.A.2.C)
- Use variables to represent numbers and write expressions when solving a real-world or mathematical problem. (6.EE.B.6)
- Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. (6.EE.B.7)
- Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. (6.EE.B.8)
- Use variables to represent two quantities in a real-world problem that change in relationship to one another. (6.EE.C.9)

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)	
OPERATIONS AND ALGEBRAIC THINKING (OA)	
WRITE AND INTERPRET NUMERICAL EXPRESSIONS	
5.OA.A.1	Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.
5.OA.A.2	Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.
EXPRESSIONS AND EQUATIONS (EE)	
APPLY AND EXTEND PREVIOUS UNDERSTANDINGS OF ARITHMETIC TO ALGEBRAIC EXPRESSIONS	
6.EE.A.1	Write and evaluate numerical expressions involving whole-number exponents.
6.EE.A.2	Write, read, and evaluate expressions in which letters stand for numbers.
6.EE.A.2.A	Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation "Subtract y from 5" as 5 - y.
6.EE.A.2.B	Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.
6.EE.A.2.C	Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas V = s3 and A = 6 s2 to find the volume and surface area of a cube with sides of length s = 1/2.
6.EE.A.3	Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.
6.EE.A.4	Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.

continues

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)

REASON ABOUT AND SOLVE ONE-VARIABLE EQUATIONS AND INEQUALITIES	
6.EE.B.5	Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
6.EE.B.6	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
6.EE.B.7	Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.
6.EE.B.8	Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.
REPRESENT AND ANALYZE QUANTITATIVE RELATIONSHIPS BETWEEN DEPENDENT AND INDEPENDENT VARIABLES	
6.EE.C.9	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.

NOTES

CHAPTER 8

PROBLEM SOLVING

PROBLEM SOLVING

SKILLS INVENTORY

Lower Elementary

Uses the four methods of problem solving (sensorial, modeling, arithmetic, algebraic) to solve real world and mathematical problems dealing with whole number operations, measurement (money, length, time, mass, volume) and geometry (area and perimeter of plane figures).

Upper Elementary

Uses the four methods of problem solving (sensorial, modeling, arithmetic, algebraic) to solve real world and mathematical problems dealing with whole number operations, fraction operations, measurement (distance, time, volume, mass, money), geometry (area and perimeter of plane figures), volume, algebra, ratio, proportion, percent, and coordinate systems.

MONTESSORI LESSONS	PURPOSES
All Levels These methods are used at the initial, middle and later series with increasing complexity	
Solving Problems Sensorially	To use Montessori materials for solving word problems.
Solving Problems by Modeling	To use models (pictures, graphs, diagrams, words) to help solve word problems.
Solving Problems Arithmetically	To solve word problems using paper and pencil.
Solving Problems Algebraically	To become familiar with formulas.

continues

MONTESSORI LESSONS	PURPOSES
INITIAL SERIES	
SOLVING PROBLEMS SENSORIALLY AND SOLVING PROBLEMS BY MODELING	
Addition and Subtraction	• To solve addition and subtraction word problems using objects with a symbol for the unknown number to represent the problem. • To solve word problems that call for addition of three whole numbers. • To complete addition and subtraction word problems with unknowns in all positions. • To use addition and subtraction to solve one- and two-step word problems.
Measurement	• To solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. • To use addition and subtraction to solve word problems involving lengths that are given in the same units.
SOLVING PROBLEMS ARITHMETICALLY	
Addition and Subtraction	• As above.
Measurement	• As above. • To use equations with a symbol for the unknown number to represent the length word problem.
SOLVING PROBLEMS ALGEBRAICALLY	
Addition and Subtraction	• To solve addition and subtraction word problems with a symbol for the unknown number to represent the problem. • To complete addition and subtraction word problems with unknowns in all positions. • To use addition and subtraction to solve one- and two-step word problems.
Measurement	• To use a symbol for the unknown number to represent the length word problem.
Vocabulary	• To determine if addition and subtraction should be used based on the language of adding to, taking from, putting together, taking apart, and comparing within word problems.

continues

MONTESSORI LESSONS	PURPOSES
MIDDLE SERIES	
SOLVING PROBLEMS SENSORIALLY	
Fraction Operations	• To use visual fraction models to represent addition and subtraction word problems. • To use visual fraction models to represent multiplication word problems.
SOLVING PROBLEMS MODELING	
Whole Number Operations	• To solve multiplication and division word problems using drawings with a symbol for the unknown number to represent the problem.
Measurement	• To represent addition and subtraction time word problems on a number line diagram. • To use drawings (such as a beaker with a measurement scale) to represent mass and volume word problems. • To represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
SOLVING PROBLEMS ARITHMETICALLY	
Whole Number Operations	• To solve whole number multi step word problems using the four operations, including division problems in which remainders must be interpreted. • To assess the reasonableness of word problem answers using mental computation and estimation strategies including rounding. • To solve word problems in situations involving equal groups, arrays, and measurement quantities.
Fraction Operations	• To solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators. • To use fraction equations to represent addition and subtraction word problems. • To solve word problems involving multiplication of a fraction by a whole number. • To use equations to represent multiplication word problems.

continues

MONTESSORI LESSONS	PURPOSES
Measurement	• To solve word problems involving addition and subtraction of time intervals in minutes. • To add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units. • To use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals.
Geometry	• To solve real world and mathematical problems involving perimeters of polygons (find the perimeter given the side lengths, find an unknown side length). • To find rectangles with the same perimeter and different areas or with the same area and different perimeters. • To apply the area and perimeter formulas for rectangles in real world and mathematical problems.
SOLVING PROBLEMS ALGEBRAICALLY	
Whole Number Operations	• To represent multi step word problems using equations with a letter standing for the unknown quantity. • To solve multiplication and division word problems using equations with a symbol for the unknown number to represent the problem. • To multiply or divide to solve word problems involving multiplicative comparison, distinguishing multiplicative comparison from additive comparison.
LATER SERIES	
SOLVING PROBLEMS SENSORIALLY	
Fraction Operations	• To use visual fraction models or equations to represent addition and subtraction fraction word problems. • To use visual fraction models to represent multiplication fraction word problems. • To use visual fraction models to represent division fraction word problems.
Area and Perimeter	• To solve word problems by finding the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes.

continues

MONTESSORI LESSONS	PURPOSES
SOLVING PROBLEMS MODELING	
Algebra	• To represent solutions of inequality word problems on number line diagrams. • To analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.
Ratio, Proportion, & Percentage	• To reason about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations to solve word problems.
Area and Perimeter	• To solve real world problems by representing three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures.
SOLVING PROBLEMS ARITHMETICALLY	
Fraction Operations	• To solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators. • To use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. • To use fraction equations to represent fraction word problems. • To solve real world problems involving multiplication of fractions and mixed numbers. • To use fraction equations to represent division fraction word problems. • To solve real world problems involving division of unit fractions by non-zero whole numbers. • To solve real world problems involving division of whole numbers by unit fractions.
Fraction Operations	• To use ratio and rate reasoning to solve real-world and mathematical problems. • To solve unit rate problems including those involving unit pricing and constant speed. • To solve problems finding the whole, given a part and the percent.

continues

MONTESSORI LESSONS	PURPOSES
Coordinate Systems	• To represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. • To solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane.
Volume	• To solve real world and mathematical problems involving volume.
SOLVING PROBLEMS ALGEBRAICALLY	
Algebra	• To solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. • To write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. • To recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions. • To use variables to represent two quantities in a real-world problem that change in relationship to one another. • To write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable.
Volume	• To apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

ASSESSMENT VOCABULARY

INITIAL SERIES

add
addition
compare
equal
equation
less than (<)
one-step problem
put together
represent
subtraction
sum
symbol
take apart
take from
three
twenty
two-step problem
unknown
whole number
word problem

Cognitive Verbs

compare
represent
solve

MIDDLE SERIES

In addition to previous vocabulary:

additive
area
array
denominator
divide
division
estimation
four
fraction
group/grouping
mathematical problem
measurement
mental math
multiplication
multiply
multistep problem
operation
perimeter
polygon
quantity
real-world problem
reasonable
rectangle
remainder
rounding
side length
strategy
visual fraction model
whole

Cognitive Verbs

answer
assess
distinguish
interpret
pose
refer

LATER SERIES

In addition to previous vocabulary:

absolute value
associative property
base
benchmark fraction
compose
constraint
coordinate
coordinate plane
decompose/ decomposition
dependent variable
distance
double number line diagram
edge length
equivalent
first coordinate
first quadrant
formula
graph
height
independent variable
inequality
infinitely many
mixed number
net
number line
one
point
prism
product
quadrant
quotient
rate
ratio
rational number
real-world problem
reasonable
rectangle
rectangular prism
relationship
right triangle
second coordinate
shape
solution
special quadrilateral
surface area
table
tape diagram
three dimensional
triangle
two
unit cube
unit fraction
unlike denominators
variable
volume
whole

Cognitive Verbs

analyze
answer
apply
compose
compute
decompose
estimate
express
form
graph
reason
recognize
relate
represent

ASSESSMENT CONSIDERATIONS

INITIAL SERIES

Students will be asked to:

Whole Number Operations: Addition

- Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20. (1.OA.A.2)
- Use objects, drawings, and equations with a symbol for the unknown number to represent the problem. (1.OA.A.2)

Whole Number Operations: Addition and Subtraction

- Use addition and subtraction within 20 to solve word problems. (1.OA.A.1)
- Determine addition and subtraction based on the language of adding to, taking from, putting together, taking apart, and comparing within word problems. (1.OA.A.1, (2.OA.A.1)
- Complete addition and subtraction word problems with unknowns in all positions. (1.OA.A.1, 2.OA.A.1)
- Solve addition and subtraction word problems using objects, drawings, and equations with a symbol for the unknown number to represent the problem. (1.OA.A.1, 2.OA.A.1)
- Use addition and subtraction within 100 to solve one- and two-step word problems. (2.OA.A.1)

Measurement

- Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. (2.MD.C.8)
- Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units. (2.MD.B.5)
- Use drawings and equations with a symbol for the unknown number to represent the length word problem. (2.MD.B.5)

MIDDLE SERIES

Students will be asked to:

Whole Number Operations: Addition and Subtraction

- Solve two-step word problems using the four operations. (3.OA.D.8)
- Represent two-step word problems using equations with a letter standing for the unknown quantity. (3.OA.D.8)
- Solve whole number multi step word problems using the four operations. (4.OA.A.3)
- Represent multi step word problems using equations with a letter standing for the unknown quantity. (4.OA.A.3)
- Assess the reasonableness of word problem answers using mental computation and estimation strategies including rounding. (3.OA.D.8, 4.OA.A.3)

continues

ASSESSMENT CONSIDERATIONS

Whole Number Operations: Multiplication and Division

- Use multiplication and division within 100 to solve word problems. (3.OA.A.3)
- Solve word problems in situations involving equal groups, arrays, and measurement quantities. (3.OA.A.3)
- Solve two-step word problems using the four operations. (3.OA.D.8)
- Represent two-step word problems using equations with a letter standing for the unknown quantity. (3.OA.D.8)
- Multiply or divide to solve word problems involving multiplicative comparison, distinguishing multiplicative comparison from additive comparison. (4.OA.A.2)
- Solve multiplication and division word problems using drawings, and equations with a symbol for the unknown number to represent the problem. (4.OA.A.2)
- Solve whole number multi step word problems using the four operations, including division problems in which remainders must be interpreted. (4.OA.A.3)
- Represent multi step word problems using equations with a letter standing for the unknown quantity. (4.OA.A.3)
- Assess the reasonableness of word problem answers using mental computation and estimation strategies including rounding. (3.OA.D.8, 4.OA.A.3)

Fractions: Addition and Subtraction

- Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators. (4.NF.B.3.D)
- Use visual fraction models and equations to represent addition and subtraction word problems. (4.NF.B.3.D)

Fractions: Multiplication

- Solve word problems involving multiplication of a fraction by a whole number. (4.NF.B.4.C)
- Use visual fraction models and equations to represent multiplication word problems. (4.NF.B.4.C)

Measurement

- Solve word problems involving addition and subtraction of time intervals in minutes. (3.MD.A.1)
- Represent addition and subtraction time word problems on a number line diagram. (3.MD.A.1)
- Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units. (3.MD.A.2)
- Use drawings (such as a beaker with a measurement scale) to represent mass and volume word problems. (3.MD.A.2)
- Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals. (4.MD.A.2)
- Express measurements given in a larger unit in terms of a smaller unit. (4.MD.A.2)
- Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. (4.MD.A.2)

Area and Perimeter of Plane Figures

- Solve real world and mathematical problems involving perimeters of polygons (find the perimeter given the side lengths, find an unknown side length). (3.MD.D.8)
- Find rectangles with the same perimeter and different areas or with the same area and different perimeters. (3.MD.D.8)
- Apply the area and perimeter formulas for rectangles in real world and mathematical problems. (4.MD.A.3)

continues

ASSESSMENT CONSIDERATIONS

LATER SERIES

Students will be asked to:

Fractions: Addition and Subtraction

- Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators. (5.NF.A.2)
- Use visual fraction models or equations to represent fraction word problems. (5.NF.A.2)
- Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. (5.NF.A.2)

Fractions: Multiplication

- Solve real world problems involving multiplication of fractions and mixed numbers. (5.NF.B.6)
- Use visual fraction models or equations to represent fraction word problems. (5.NF.B.6)

Fractions: Division

- Solve real world problems involving division of unit fractions by non-zero whole numbers. (5.NF.B.7.C)
- Solve real world problems involving division of whole numbers by unit fractions. (5.NF.B.7.C)
- Use visual fraction models or equations to represent fraction word problems. (5.NF.B.7.C)

Algebra

- Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers. (6.EE.B.7)
- Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. (6.EE.B.8)
- Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions. (6.EE.B.8)
- Represent solutions of inequality word problems on number line diagrams. (6.EE.B.8)
- Use variables to represent two quantities in a real-world problem that change in relationship to one another. (6.EE.C.9)
- Write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. (6.EE.C.9)
- Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. (6.EE.C.9)

Ratio, Proportion and Percentage

- Use ratio and rate reasoning to solve real-world and mathematical problems. (6.RP.A.3)
- Reason about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations to solve word problems. (6.RP.A.3)
- Solve unit rate problems including those involving unit pricing and constant speed. (6.RP.A.3.B)
- Solve problems finding the whole, given a part and the percent. (6.RP.A.3.C)

Coordinate Systems

- Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. (5.G.A.2)
- Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. (6.NS.C.8)

continues

ASSESSMENT CONSIDERATIONS

Area and Perimeter of Plane Figures

- Solve word problems by finding the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes. (6.G.A.1)
- Solve real world problems by representing three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. (6.G.A.4)

Volume

- Solve real world and mathematical problems involving volume. (5.MD.C.5)
- Apply the formulas V = l w h and V = b h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. (6.G.A.2)

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)

OPERATIONS AND ALGEBRAIC THINKING (OA)

REPRESENT AND SOLVE PROBLEMS INVOLVING ADDITION AND SUBTRACTION

1.OA.A.1	Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
1.OA.A.2	Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
2.OA.A.1	Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

REPRESENT AND SOLVE PROBLEMS INVOLVING MULTIPLICATION AND DIVISION

3.OA.A.3	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

continues

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)	
SOLVE PROBLEMS INVOLVING THE FOUR OPERATIONS, AND IDENTIFY AND EXPLAIN PATTERNS IN ARITHMETIC	
3.OA.D.8	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
USE THE FOUR OPERATIONS WITH WHOLE NUMBERS TO SOLVE PROBLEMS	
4.OA.A.2	Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.
4.OA.A.3	Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
NUMBER AND OPERATIONS: FRACTIONS (NF)	
BUILD FRACTIONS FROM UNIT FRACTIONS	
4.NF.B.3.D	Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
4.NF.B.4.C	Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?
USE EQUIVALENT FRACTIONS AS A STRATEGY TO ADD AND SUBTRACT FRACTIONS	
5.NF.A.2	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + ½ = 3/7, by observing that 3/7 < ½.

continues

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)	
APPLY AND EXTEND PREVIOUS UNDERSTANDINGS OF MULTIPLICATION AND DIVISION	
5.NF.B.6	Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
5.NF.B.7.C	Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share ½ lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?
MEASUREMENT AND DATA (MD)	
RELATE ADDITION AND SUBTRACTION TO LENGTH	
2.MD.B.5	Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.
WORK WITH TIME AND MONEY	
2.MD.C.8	Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have?
SOLVE PROBLEMS INVOLVING MEASUREMENT AND ESTIMATION	
3.MD.A.1	Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.
3.MD.A.2	Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).1 Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.
GEOMETRIC MEASUREMENT: RECOGNIZE PERIMETER	
3.MD.D.8	Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

continues

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)	
SOLVE PROBLEMS INVOLVING MEASUREMENT AND CONVERSION OF MEASUREMENTS	
4.MD.A.2	Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
4.MD.A.3	Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.
GEOMETRIC MEASUREMENT: UNDERSTAND CONCEPTS OF VOLUME	
5.MD.C.5	Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.
GEOMETRY (G)	
GRAPH POINTS ON THE COORDINATE PLANE TO SOLVE REAL-WORLD AND MATHEMATICAL PROBLEMS	
5.G.A.2	Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.
SOLVE REAL-WORLD AND MATHEMATICAL PROBLEMS INVOLVING AREA, SURFACE AREA, AND VOLUME	
6.G.A.1	Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
6.G.A.2	Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = l w h and V = b h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
6.G.A.4	Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

continues

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)	
RATIOS AND PROPORTIONAL RELATIONSHIPS (RP)	
UNDERSTAND RATIO CONCEPTS AND USE RATIO REASONING TO SOLVE PROBLEMS	
6.RP.A.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
6.RP.A.3.B	Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?
6.RP.A.3.C	Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
THE NUMBER SYSTEM (NS)	
APPLY AND EXTEND PREVIOUS UNDERSTANDINGS OF NUMBERS TO THE SYSTEM OF RATIONAL NUMBERS	
6.NS.C.8	Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.
EXPRESSIONS AND EQUATIONS (EE)	
REASON ABOUT AND SOLVE ONE-VARIABLE EQUATIONS AND INEQUALITIES	
6.EE.B.7	Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.
6.EE.B.8	Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.
REPRESENT AND ANALYZE QUANTITATIVE RELATIONSHIPS BETWEEN DEPENDENT AND INDEPENDENT VARIABLES	
6.EE.C.9	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.

NOTES

CHAPTER 9

RATIO, PROPORTION, AND PERCENT

RATIO, PROPORTION, AND PERCENT

SKILLS INVENTORY
Upper Elementary Understands the concept of ratio, including proportion and percentage, and uses language to describe a ratio relationship between quantities.

MONTESSORI LESSONS	PURPOSES
LATER SERIES	
Ratio • Concept, Language, Notation of Ratio • Ratio can be Expressed as a Fraction • Ratio Expresses a Division • Equal Ratios	• To discover that ratios, fractions, and division are all ways of looking at the same concepts. • To understand the concept of a ratio. • To use ratio language to describe a ratio relationship between two quantities. • To make tables of equivalent ratios relating quantities with whole-number measurements. • To find missing values in the tables of equivalent ratios. • To use tables to compare ratios. • To use ratio reasoning to convert measurement units.
Proportion • Concept, Language, Notation of Proportion • Cross Multiplication	• To recognize that a proportion is a claim that two ratios are equal. • To recognize examples of proportion in everyday life.
Percent • Centesimal Frame • Concept (hundredths), language, notation • Conversion of fraction insets to percentage	• To find a percent of a quantity as a rate per 100. • To understand a percentage is another way of describing a ratio with respect to 100.

continues

MONTESSORI LESSONS	PURPOSES
Problem Solving • Teacher-Created or Purchased Cards • Experiences in the Classroom	• To use ratio and rate reasoning to solve real-world and mathematical problems. • To solve unit rate problems including those involving unit pricing and constant speed. • To solve problems involving finding the whole, given a part and the percent.

ASSESSMENT VOCABULARY

LATER SERIES

associate	rate	**Cognitive Verbs**
double number line diagram	ratio	associate
equation	real-world problem	describe
equivalent	table	reason
mathematical problem	tape diagram	solve
percent	two	understand
quantity	unit rate	

ASSESSMENT CONSIDERATIONS

Students will be asked to:

- Understand the concept of a ratio. (6.RP.A.1)
- Use ratio language to describe a ratio relationship between two quantities. (6.RP.A.1)
- Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0. (6.RP.A.2)
- Use rate language in the context of a ratio relationship. (6.RP.A.2)
- Make tables of equivalent ratios relating quantities with whole-number measurements. (6.RP.A.3.A)
- Find missing values in the tables of equivalent ratios. (6.RP.A.3.A)
- Plot the pairs of values on the coordinate plane. (6.RP.A.3.A)
- Use tables to compare ratios. (6.RP.A.3.A)
- Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity. (6.RP.A.3.C)
- Use ratio reasoning to convert measurement units. (6.RP.A.3.D)
- Manipulate and transform measurement units when multiplying or dividing quantities. (6.RP.A.3.D)

Word Problems

- Use ratio and rate reasoning to solve real-world and mathematical problems. (6.RP.A.3)
- Solve unit rate problems including those involving unit pricing and constant speed. (6.RP.A.3.B)
- Solve problems involving finding the whole, given a part and the percent. (6.RP.A.3.C)

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)	
RATIOS AND PROPORTIONAL RELATIONSHIPS (RP)	
UNDERSTAND RATIO CONCEPTS AND USE RATIO REASONING TO SOLVE PROBLEMS	
6.RP.A.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."
6.RP.A.2	Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is ¾ cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."1
6.RP.A.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
6.RP.A.3.A	Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
6.RP.A.3.B	Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?
6.RP.A.3.C	Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
6.RP.A.3.D	Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

NOTES

CHAPTER 10

MEASUREMENT AND DATA

DATA AND GRAPHS

SKILLS INVENTORY

Lower Elementary

Collects and organizes data, uses a variety of graph types (line plots, picture graphs, bar graphs, scaled picture graphs, scaled bar graphs, line plots, pie graphs) to represent data and is able to interpret data.

Upper Elementary

Collects and organizes data, uses a variety of graph types (line plot, dot plot, histogram, box plot, line graph) to represent data and is able to summarize and describe distributions, including an understanding of statistical variability.

MONTESSORI LESSONS	PURPOSES
INITIAL SERIES	
Collecting, Organizing, and Interpreting Data • Teacher-Created Lessons	• To learn to collect and work with different types of data. • To generate measurement data by measuring lengths of several objects to the nearest whole unit. • To generate measurement data by making repeated measurements of the same object. • To learn to work with different types of data. • To organize, represent, and interpret data • To ask and answer questions about the total number of data points.
Picture Graph	• To practice interpreting and constructing picture graphs. • To draw a picture graph to represent a data set.
Bar Graph	• To practice interpreting and constructing bar graphs. • To draw a bar graph to represent a data set. • To solve simple, put-together, take-apart, and compare problems using information presented in a bar graph.
Line Plot	• To practice interpreting and constructing line plots. • To show measurements by making a line plot, where the horizontal scale is marked off in whole-number units.

continues

MONTESSORI LESSONS	PURPOSES
MIDDLE SERIES	
Collecting, Organizing, and Interpreting Data • Teacher-Created Lessons	• To generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch.
Scaled Picture Graph	• To practice interpreting and constructing scaled picture graph. • To draw a scaled picture graph to represent a data set with several categories.
Scaled Bar Graph	• To practice interpreting and constructing scaled bar graph. • To draw a scaled bar graph to represent a data set with several categories. • To solve one- and two-step "How many more?" and "How many less?" problems using information presented in scaled bar graphs.
Line Plot	• To practice interpreting and constructing scaled line plots. • To show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters. • To make a line plot to display a data set of measurements in fractions of a unit (1/2, ¼, 1/8). • To solve problems involving addition and subtraction of fractions by using information presented in line plots.
Pie Graph	• To practice interpreting and constructing scaled pie graphs. • To draw a pie graph to represent a data set with several categories.

continues

MONTESSORI LESSONS	PURPOSES
LATER SERIES	
Collecting, Organizing, and Interpreting Data • Teacher-Created Lessons	• To recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. • To understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. • To recognize that a measure of center for a numerical data set summarizes all of its values with a single number. • To recognize that a measure of variation describes how its values vary with a single number.
Summarize Numerical Data Sets • Teacher-Created Lessons	• To report the number of observations. • To describe the nature of the attribute under investigation, including how it was measured and its units of measurement. • To give quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation). • To describe any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. • To relate the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
Line Graph	• To practice interpreting and constructing line graphs.
Line Plot	• To practice interpreting and constructing line plots. • To make a line plot to display a data set of measurements in fractions of a unit (1/2, ¼, 1/8). • To use operations on fractions to solve problems involving information presented in line plots.
Dot Plot	• To practice interpreting and constructing dot plots. • To display numerical data in plots on a number line, including dot plots.
Histogram	• To practice interpreting and constructing histograms. • To display numerical data in plots on a number line, including histograms.
Box Plot	• To practice interpreting and constructing box plots. • To display numerical data in plots on a number line, including box plots.

ASSESSMENT VOCABULARY

INITIAL SERIES	MIDDLE SERIES	LATER SERIES
category data data point horizontal scale how many length less line plot measurement more number one represent scale three total unit whole number **Cognitive Verbs** answer ask generate interpret measure organize present represent solve	*In addition to previous vocabulary:* addition bar graph data set display fourth (fraction) fraction half inch (in) information one-step problem picture graph quarter (one-fourth) ruler scaled graph subtraction two-step problem	*In addition to previous vocabulary:* attribute box plot center data distribution deviation distribution dot plot fraction histogram interquartile range mean mean absolute deviation measures of center/central tendency median number line observation operation pattern plot quantitative set shape spread statistical question striking deviation value variability variation **Cognitive Verbs** anticipate describe gather recognize relate report summarize understand

ASSESSMENT CONSIDERATIONS

INITIAL SERIES

Students will be asked to:

- Create graphs:
  - Line Plot
  - Picture Graph
  - Bar Graph
- Organize, represent, and interpret data with up to three categories. (1.MD.C.4)
- Ask and answer questions about the total number of data points, how many are in each category, and how many more or less are in one category than in another. (1.MD.C.4)
- Generate measurement data by measuring lengths of several objects to the nearest whole unit. (2.MD.D.9)
- Generate measurement data by making repeated measurements of the same object. (2.MD.D.9)
- Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. (2.MD.D.9)
- Draw a picture graph and a bar graph to represent a data set with up to four categories. (2.MD.D.10)
- Solve simple put-together, take-apart, and compare problems using information presented in a bar graph. (2.MD.D.10)

MIDDLE SERIES

Students will be asked to:

- Create graphs:
  - Scaled Picture Graph
  - Scaled Bar Graph
  - Line Plot
  - Pie Graph
- Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. (3.MD.B.3)
- Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. (3.MD.B.3)
- Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. (3.MD.B.4)
- Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters. (3.MD.B.4)
- Make a line plot to display a data set of measurements in fractions of a unit (1/2, ¼, 1/8). (4.MD.B.4)
- Solve problems involving addition and subtraction of fractions by using information presented in line plots. (4.MD.B.4)

continues

ASSESSMENT CONSIDERATIONS

LATER SERIES

Students will be asked to:

- Create graphs:
  - Line Plot
  - Dot Plot
  - Histogram
  - Box Plot
  - Line Graph
- Make a line plot to display a data set of measurements in fractions of a unit (1/2, ¼, 1/8). (5.MD.B.2)
- Use operations on fractions for this grade to solve problems involving information presented in line plots. (5.MD.B.2)
- Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. (6.SP.A.1)
- Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. (6.SP.A.2)
- Recognize that a measure of center for a numerical data set summarizes all of its values with a single number. (6.SP.A.3)
- Recognize that a measure of variation describes how its values vary with a single number. (6.SP.A.3)
- Display numerical data in plots on a number line, including dot plots, histograms, and box plots. (6.SP.B.4)
- Summarize numerical data sets in relation to their context, such as by: (6.SP.B.5)
- Reporting the number of observations. (6.SP.B.5.A)
- Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. (6.SP.B.5.B)
- Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation). (6.SP.B.5.C)
- Describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. (6.SP.B.5.C)
- Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered. (6.SP.B.5.D)

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)

MEASUREMENT AND DATA (MD)

REPRESENT AND INTERPRET DATA

1.MD.C.4	Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.
2.MD.D.9	Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.

continues

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)	
2.MD.D.10	Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.
3.MD.B.3	Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.
3.MD.B.4	Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.
4.MD.B.4	Make a line plot to display a data set of measurements in fractions of a unit (1/2, ¼, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.
5.MD.B.2	Make a line plot to display a data set of measurements in fractions of a unit (1/2, ¼, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.
STATISTICS AND PROBABILITY (SP)	
DEVELOP UNDERSTANDING OF STATISTICAL VARIABILITY	
6.SP.A.1	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.
6.SP.A.2	Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
6.SP.A.3	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
SUMMARIZE AND DESCRIBE DISTRIBUTIONS	
6.SP.B.4	Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
6.SP.B.5	Summarize numerical data sets in relation to their context, such as by:
6.SP.B.5.A	Reporting the number of observations.

continues

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)

6.SP.B.5.B	Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
6.SP.B.5.C	Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
6.SP.B.5.D	Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

NOTES

Bar Graph: A graph that visually displays data using horizontal or vertical bars whose lengths are proportional to quantities they represent. It can be used when one axis cannot have a numerical scale.

Box Plot: A graph that shows the distribution of data along a number line. Quartiles divide the data into four equal parts

Dot Plot: A dot plot is a type of graphic display used to compare frequency counts within categories or groups.

Histogram: A graph showing frequency of data in which the horizontal axis represents discrete units, certain ranges, or intervals while the vertical axis represents the frequency. Often, rectangular bars are drawn with their areas proportional to the frequencies within the ranges or intervals.

Line Graph: A set of points connected by line segments. This type of graph is usually used to show a trend.

Line Plot: A graph that displays data as points above a number line or some other line of characteristics or attributes.

Picture Graph: It is a diagram that uses pictures or symbols to show data for quick understanding. In a pictogram, a

picture or symbol is used to represent a specific quantity.

Pie Graph: A circular graph that uses radii to divide a circle into sectors in such a way that the areas of the sectors are proportional to the quantities represented.

MEASUREMENT

SKILLS INVENTORY

Lower Elementary

- Measures length in standard units of inches, feet, centimeters and meters using rulers, yardsticks, meter sticks and measuring tapes.
- Measures liquid volumes and masses of objects in standard units: grams (g), kilograms (kg), and liters (l).
- Works with time including hours and half-hours, nearest five minutes, a.m. and p.m.
- Works with money including dollar bills, quarters, dimes, nickels, and pennies.
- Solves word problems and real world applications for length, money, time, volume and mass.

Upper Elementary

- Understands relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec.
- Measures distances, intervals of time, liquid volumes, masses of objects, and money.
- Converts among different-sized standard measurement units within a given measurement system.
- Solves word problems and real world applications for length, volume, mass, temperature, time, money and conversion.

MONTESSORI LESSONS	PURPOSES
INITIAL SERIES	
LENGTH	
History of the Measurement of Length	• To introduce the history behind measurement. • To help the child appreciate the need for standard units of measure. • To help the child feel gratitude for the people who have solved these challenges in the past.
The Concept of Measurement	• To understand that measuring requires counting repetitions of a single unit. • To order objects by length. • To compare the lengths of two objects indirectly by using a third object. • To express the length of an object as a whole number of length units, by laying multiple copies of a shorter object end to end. • To understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.

continues

MONTESSORI LESSONS	PURPOSES
Standard Unit of Measurement for Length • Rulers • Yard Sticks • Meter Sticks • Tape Measures	• For children to use different tools to measure length. • To measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. • To measure the length of an object twice, using length units of different lengths for the two measurements, describe how the two measurements relate to the size of the unit chosen. • To estimate lengths using units of inches, feet, centimeters, and meters.
Application	• To represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, … • To represent whole-number sums and differences within 100 on a number line diagram. • To measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.
Problem Solving • Teacher-Created or Purchased Cards • Experiences in the Classroom	• To offer an opportunity to apply the knowledge of length to real-life situations. • To use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units. • To use addition and subtraction within 100 to solve word problems involving lengths with a symbol for the unknown number to represent the problem.
MONEY	
Names of Coins • Counting Money	• To associate the names and shapes/colors of coins. • To count the value of coins.
Problem Solving • Teacher-Created or Purchased Cards • Experiences in the Classroom	• To offer an opportunity to apply the knowledge of money to real-life situations. • To solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately.

continues

MONTESSORI LESSONS	PURPOSES
TIME	
Clock Material • Hours • Minutes (five) • half hours	• To expose the children to nomenclature: decade, century, millennium, millisecond, microsecond, nanosecond. • To introduce some of the ways that human beings have found to measure the passing time. • To tell and write time in hours and half-hours using an analog or digital clock. • To tell and write time from an analog or digital clock to the nearest five minutes, using a.m. and p.m.
See **History and Political Geography: Time** for more lessons on Time	
MIDDLE SERIES	
LENGTH	
The Metric System • Introduction to the Customary/ English Units of Measurement	• To provide the child with the language and experience for understanding the metric system.
Problem Solving • Teacher-Created or Purchased Cards • Experiences in the Classroom	• To use the four operations to solve word problems involving distances. • To solve problems that require expressing measurements given in a larger unit in terms of a smaller unit. • To represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
VOLUME AND MASS	
Introduction	• To provide the child with the language and experience for understanding volume and mass.
Standard Units • Grams (g) • Kilograms (kg) • Liters (l)	• To measure and estimate liquid volumes using standard units of liters (l). • To measure and estimate masses of objects using standard units of grams (g), and kilograms (kg).

continues

MONTESSORI LESSONS	PURPOSES
Problem Solving • Teacher-Created or Purchased Cards • Experiences in the Classroom	• To offer an opportunity to apply the knowledge of mass to real-life situations. • To add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units. • To use the four operations to solve word problems involving liquid volumes and masses of objects. • To solve problems that require expressing measurements given in a larger unit in terms of a smaller unit. • To represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
TEMPERATURE	
The Story of Gabriel Fahrenheit	• To foster appreciation for those who invented tools to understand the world around us.
The Metric System • Customary Units	• To read a thermometer. • To understand that the majority of the world uses the Celsius scale for measurement.
Problem Solving • Teacher-Created or Purchased Cards • Experiences in the Classroom	• To solve problems that require expressing measurements given in a larger unit in terms of a smaller unit. • To represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
TIME	
Clock Material • Hours • Minutes • Seconds • ¼ and ½ hours	• To tell and write time to the nearest minute and measure time intervals in minutes.
Application	• To represent time on a number line diagram.
Problem Solving • Teacher-Created or Purchased Cards • Experiences in the Classroom	• To offer an opportunity to apply the knowledge of time to real-life situations. • To solve word problems involving addition and subtraction of time intervals in minutes. • To use the four operations to solve word problems involving intervals of time. • To solve problems that require expressing measurements given in a larger unit in terms of a smaller unit. • To represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

continues

MONTESSORI LESSONS	PURPOSES
MONEY	
Problem Solving • Teacher-Created or Purchased Cards • Experiences in the Classroom	• To use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money. • To solve problems that require expressing measurements given in a larger unit in terms of a smaller unit. • To represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
CONVERSION	
Conversion Table	• To express measurements in a larger unit in terms of a smaller unit, within a single system of measurement. • To record measurement equivalents in a two-column table. • To generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36).
Problem Solving • Teacher-Created or Purchased Cards • Experiences in the Classroom	• To offer an opportunity to apply the knowledge of measurement to real-life situations. • To solve problems that require expressing measurements given in a larger unit in terms of a smaller unit. • To represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
LATER SERIES	
Conversion between Metric System Units and English System Units	• To prepare the child to function in a world with different measuring units. • To convert among different-sized standard measurement units within a given measurement system.
Problem Solving • Teacher-Created or Purchased Cards • Experiences in the Classroom	• To offer an opportunity to apply the knowledge of conversion to real-life situations. • To use conversions in solving multi-step, real world problems.

ASSESSMENT VOCABULARY

INITIAL SERIES

addition
bar graph
category
centimeter (cm)
compare
data set
difference
dime
dollar bill
equal
equation
estimation
foot (ft)
four
inch (in)
information
length
longer
measurement
measuring tape
meter (m)
meter stick
nickel
number
number line
one
penny
picture graph
point
put together
quarter (coin)
quarter (one-fourth)
represent
ruler
scale
shorter
size
standard unit
subtraction
sum
symbol
third (ordinal number)
three
two
unit
unknown
whole number
word problem
yardstick
zero

Cognitive Verbs

compare
describe
determine
estimate
express
measure
relate
represent
select
solve
understand

MIDDLE SERIES

In addition to previous vocabulary:

add
beaker
centimeter (cm)
decimal
diagram
distance
divide
equivalent
gram (g)
hour (hr)
kilogram (kg)
kilometer (km)
large/larger
liter (l)
mass
measurement scale
measurement unit
milliliter (ml)
minute (min)
money
multiply
one-step problem
operation
ounce (oz)
pound (lb)
quantity
second (sec)
simple fraction
small/smaller
subtract
time
time interval
two-column table
volume

Cognitive Verbs

diagram

LATER SERIES

In addition to previous vocabulary:

conversion
convert
multi step problem
real-world problem

Cognitive Verbs

convert

ASSESSMENT CONSIDERATIONS

INITIAL SERIES

Students will be asked to:

Length

- Order three objects by length. (1.MD.A.1)
- Compare the lengths of two objects indirectly by using a third object. (1.MD.A.1)
- Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object end to end. (1.MD.A.2)
- Understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. (1.MD.A.2)
- Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps. (1.MD.A.2)
- Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. (2.MD.A.1)
- Measure the length of an object twice, using length units of different lengths for the two measurements, describe how the two measurements relate to the size of the unit chosen. (2.MD.A.2)
- Estimate lengths using units of inches, feet, centimeters, and meters. (2.MD.A.3)
- Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard-length unit. (2.MD.A.4)
- Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, … (2.MD.B.6)
- Represent whole-number sums and differences within 100 on a number line diagram. (2.MD.B.6)

Time

- Tell and write time in hours and half-hours using an analog clock. (1.MD.B.3)
- Tell and write time in hours and half-hours using a digital clock. (1.MD.B.3)
- Tell and write time from an analog clock to the nearest five minutes, using a.m. and p.m. (2.MD.C.7)
- Tell and write time from a digital clock to the nearest five minutes, using a.m. and p.m. (2.MD.C.7)

Word Problems

- Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. (2.MD.C.8)
- Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units. (2.MD.B.5)
- Use addition and subtraction within 100 to solve word problems involving lengths with a symbol for the unknown number to represent the problem. (2.MD.B.5)

continues

ASSESSMENT CONSIDERATIONS

MIDDLE SERIES

Students will be asked to:

Volume and Mass

- Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (3.MD.A.2)

Time

- Tell and write time to the nearest minute and measure time intervals in minutes. (3.MD.A.1)
- Represent time on a number line diagram. (3.MD.A.1)

Conversion

- Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. (4.MD.A.1)
- Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. (4.MD.A.1)
- Record measurement equivalents in a two-column table. (4.MD.A.1)
- Generate a conversion table for feet and inches listing the number pairs: (1, 12), (2, 24), (3, 36). (4.MD.A.1)

Word Problems

- Solve word problems involving addition and subtraction of time intervals in minutes. (3.MD.A.1)
- Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units. (3.MD.A.2)
- Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money. (4.MD.A.2)
- Solve problems involving simple fractions or decimals. (4.MD.A.2)
- Solve problems that require expressing measurements given in a larger unit in terms of a smaller unit.
- (4.MD.A.2)
- Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. (4.MD.A.2)

LATER SERIES

Students will be asked to:

Conversion

- Convert among different-sized standard measurement units within a given measurement system. (5.MD.A.1)

Word Problems

- Use conversions in solving multi-step, real world problems. (5.MD.A.1)

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)

MEASUREMENT AND DATA (MD)

DESCRIBE AND COMPARE MEASURABLE ATTRIBUTES	
1.MD.A.1	Order three objects by length; compare the lengths of two objects indirectly by using a third object.
1.MD.A.2	Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.
WORK WITH TIME AND MONEY	
1.MD.B.3	Tell and write time in hours and half-hours using analog and digital clocks.
2.MD.C.7	Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
2.MD.C.8	Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. Example: "If you have 2 dimes and 3 pennies, how many cents do you have?"
MEASURE AND ESTIMATE LENGTHS IN STANDARD UNITS	
2.MD.A.1	Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
2.MD.A.2	Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
2.MD.A.3	Estimate lengths using units of inches, feet, centimeters, and meters.
2.MD.A.4	Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.
RELATE ADDITION AND SUBTRACTION TO LENGTH	
2.MD.B.5	Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.
2.MD.B.6	Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

continues

COMMON CORE STATE STANDARDS (CCSS.MATH.CONTENT)

SOLVE PROBLEMS INVOLVING MEASUREMENT AND ESTIMATION

3.MD.A.1	Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.
3.MD.A.2	Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).1 Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.

SOLVE PROBLEMS INVOLVING MEASUREMENT AND CONVERSION OF MEASUREMENTS

4.MD.A.1	Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...
4.MD.A.2	Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

CONVERT LIKE MEASUREMENT UNITS WITHIN A GIVEN MEASUREMENT SYSTEM

5.MD.A.1	Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

NOTES

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His.3.K-2	Generate questions about individuals and groups who have shaped a significant historical change.	**Foundations** • The Story of Numbers
His.3.3-5	Generate questions about individuals and groups who have shaped significant historical changes and continuities.	

COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS CCSS.ELA.LITERACY		MONTESSORI CHAPTERS AND SECTIONS
SPEAKING AND LISTENING (SL)		
COMPREHENSION AND COLLABORATION		
SL.1.3	Ask and answer questions about what a speaker says in order to gather additional information or clarify something that is not understood.	**Foundations** • The Story of Numbers
SL.1.2	Ask and answer questions about key details in a text read aloud or information presented orally or through other media.	
SL.2.3	Ask and answer questions about what a speaker says in order to clarify comprehension, gather additional information, or deepen understanding of a topic or issue.	
SL.2.2	Recount or describe key ideas or details from a text read aloud or information presented orally or through other media.	

continues

COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS CCSS.MATH.PRACTICE		MONTESSORI CHAPTERS AND SECTIONS
MATHEMATICAL PRACTICE (MP)		
PROBLEM SOLVING		
MP1	Make sense of problems and persevere in solving them.	**Mathematical Practice** • Human Tendencies and the Mathematical Mind **Problem Solving** • Problem Solving
MP6	Attend to precision.	
COMMUNICATION: REASONING AND EXPLAINING		
MP2	Reason abstractly and quantitatively.	**Mathematical Practice** • Communication: Reasoning and Explaining
MP3	Construct viable arguments and critique the reasoning of others.	
MODELLING AND USING TOOLS		
MP4	Model with mathematics.	**Mathematical Practice** • Modelling and Using Tools
MP5	Use appropriate tools strategically.	
SEEING STRUCTURES AND GENERALIZING		
MP7	Look for and make use of structure.	**Mathematical Practice** • Seeing Structures and Generalizing
MP8	Look for and express regularity in repeated reasoning.	
CCSS.MATH.CONTENT		
OPERATIONS AND ALGEBRAIC THINKING (OA)		
REPRESENT AND SOLVE PROBLEMS INVOLVING ADDITION AND SUBTRACTION		
1.OA.A.1	Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem	**Whole Numbers** • Operations: Addition • Operations: Subtraction **Problem Solving** • Problem Solving

continues

COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS		MONTESSORI CHAPTERS AND SECTIONS
1.OA.A.2	Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	**Whole Numbers** • Operations: Addition • Operations: Subtraction **Problem Solving** • Problem Solving
2.OA.A.1	Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	
UNDERSTAND AND APPLY PROPERTIES OF OPERATIONS AND THE RELATIONSHIP BETWEEN ADDITION AND SUBTRACTION		
1.OA.B.3	Apply properties of operations as strategies to add and subtract.2 Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)	**Whole Numbers** • Properties of Numbers
1.OA.B.4	Understand subtraction as an unknown-addend problem. For example, subtract 10 - 8 by finding the number that makes 10 when added to 8.	**Whole Numbers** • Operations: Subtraction • Properties of Numbers
ADD AND SUBTRACT WITHIN 20		
1.OA.C.5	Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).	**Whole Numbers** • Operations: Addition • Operations: Subtraction

continues

COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS		MONTESSORI CHAPTERS AND SECTIONS
1.OA.C.6	Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 - 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).	**Whole Numbers** • Properties of Numbers
2.OA.B.2	Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.	
WORK WITH ADDITION AND SUBTRACTION EQUATIONS		
1.OA.D.7	Understand the meaning of the equal sign,and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 - 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.	**Whole Numbers** • Operations: Addition • Operations: Subtraction
1.OA.D.8	Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = _ - 3, 6 + 6 = _.	
WORK WITH EQUAL GROUPS OF OBJECTS TO GAIN FOUNDATIONS FOR MULTIPLICATION		
2.OA.C.3	Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.	**The Decimal System** • Number and Quantity
2.OA.C.4	Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.	**Whole Number** • Operations: Addition

continues

COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS		MONTESSORI CHAPTERS AND SECTIONS
REPRESENT AND SOLVE PROBLEMS INVOLVING MULTIPLICATION AND DIVISION		
3.OA.A.1	Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 × 7.	**Whole Numbers** • Operations: Multiplication
3.OA.A.2	Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.	**Whole Numbers** • Operations: Division
3.OA.A.3	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	**Whole Numbers** • Operations: Multiplication • Operations: Division **Problem Solving** • Problem Solving
3.OA.A.4	Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?	**Whole Numbers** • Operations: Multiplication • Operations: Division
UNDERSTAND PROPERTIES OF MULTIPLICATION AND THE RELATIONSHIP BETWEEN MULTIPLICATION AND DIVISION		
3.OA.B.5	Apply properties of operations as strategies to multiply and divide.2 Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)	**Whole Numbers** • Properties of Numbers

continues

COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS		MONTESSORI CHAPTERS AND SECTIONS
3.OA.B.6	Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.	**Whole Numbers** • Operations: Division
MULTIPLY AND DIVIDE WITHIN 100		
3.OA.C.7	Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.	**Whole Numbers** • Operations: Multiplication • Operations: Division • Properties of Numbers
SOLVE PROBLEMS INVOLVING THE FOUR OPERATIONS, AND IDENTIFY AND EXPLAIN PATTERNS IN ARITHMETIC=		
3.OA.D.8	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding	**Whole Numbers** • Operations: Addition • Operations: Subtraction, • Operations: Multiplication • Operations: Division **Problem Solving** • Problem Solving
3.OA.D.9	Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.	
USE THE FOUR OPERATIONS WITH WHOLE NUMBERS TO SOLVE PROBLEMS		
4.OA.A.1	Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.	**Whole Numbers** • Operations: Multiplication

continues

COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS		MONTESSORI CHAPTERS AND SECTIONS
4.OA.A.2	Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.	**Whole Numbers** • Operations: Multiplication • Operations: Division **Problem Solving** • Problem Solving
4.OA.A.3	Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	
GAIN FAMILIARITY WITH FACTORS AND MULTIPLES		
4.OA.B.4	Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.	**Whole Numbers** • Multiples, Factors and Divisibility
GENERATE AND ANALYZE PATTERNS		
4.OA.C.5	Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.	**Whole Numbers** • Operations: Addition • Operations: Subtraction • Operations: Multiplication • Operations: Division

continues

COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS		MONTESSORI CHAPTERS AND SECTIONS
ANALYZE PATTERNS AND RELATIONSHIPS		
5.OA.B.3	Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.	**Whole Numbers** • Operations: Addition • Operations: Subtraction • Operations: Multiplication • Operations: Division
WRITE AND INTERPRET NUMERICAL EXPRESSIONS		
5.OA.A.1	Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.	**Algebra** • Algebra
5.OA.A.2	Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.	
MEASUREMENT AND DATA (MD)		
MEASURE LENGTHS INDIRECTLY AND BY ITERATING LENGTH UNITS		
1.MD.A.1	Order three objects by length; compare the lengths of two objects indirectly by using a third object.	**Measurement and Data** • Measurement
1.MD.A.2	Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.	

continues

COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS		MONTESSORI CHAPTERS AND SECTIONS
TELL AND WRITE TIME		
1.MD.B.3	Tell and write time in hours and half-hours using analog and digital clocks.	**Measurement and Data** • Measurement
REPRESENT AND INTERPRET DATA		
1.MD.C.4	Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.	**Ratio, Proportion, and Percent** • Ratio, Proportion, and Percent **Measurement and Data** • Data and Graphs
2.MD.D.9	Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.	
2.MD.D.10	Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems1 using information presented in a bar graph.	
3.MD.B.3	Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.	
3.MD.B.4	Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.	
4.MD.B.4	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.	

continues

COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS		MONTESSORI CHAPTERS AND SECTIONS
5.MD.B.2	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.	**Ratio, Proportion, and Percent** • Ratio, Proportion, and Percent **Measurement and Data** • Data and Graphs
WORK WITH TIME AND MONEY		
2.MD.C.7	Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.	**Measurement and Data** • Measurement
2.MD.C.8	Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have?	**Measurement and Data** • Measurement **Problem Solving** • Problem Solving
MEASURE AND ESTIMATE LENGTHS IN STANDARD UNITS		
2.MD.A.1	Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.	**Measurement and Data** • Measurement
2.MD.A.2	Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.	
2.MD.A.3	Estimate lengths using units of inches, feet, centimeters, and meters.	
2.MD.A.4	Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.	

continues

COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS		MONTESSORI CHAPTERS AND SECTIONS
RELATE ADDITION AND SUBTRACTION TO LENGTH		
2.MD.B.5	Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.	**Measurement and Data** • Measurement **Problem Solving** • Problem Solving
2.MD.B.6	Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.	**Measurement and Data** • Measurement
SOLVE PROBLEMS INVOLVING MEASUREMENT AND ESTIMATION		
3.MD.A.1	Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.	**Measurement and Data** • Measurement **Problem Solving** • Problem Solving
3.MD.A.2	Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).1 Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem	**Measurement and Data** • Measurement
4.MD.A.1	Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...	

continues

COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS		MONTESSORI CHAPTERS AND SECTIONS
4.MD.A.2	Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	**Measurement and Data** • Measurement **Problem Solving** • Problem Solving
4.MD.A.3	Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.	**Problem Solving** • Problem Solving
CONVERT LIKE MEASUREMENT UNITS WITHIN A GIVEN MEASUREMENT SYSTEM		
5.MD.A.1	Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.	**Measurement and Data** • Measurement
GEOMETRIC MEASUREMENT: UNDERSTAND CONCEPTS OF VOLUME		
5.MD.C.5	Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.	**Problem Solving** • Problem Solving
GEOMETRY (G)		
REASON WITH SHAPES AND THEIR ATTRIBUTES		
1.G.A.3	Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.	**Fractions** • Foundations • Fractions • Equivalence

continues

COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS		MONTESSORI CHAPTERS AND SECTIONS
2.G.A.2	Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.	**Fractions** • Foundations
2.G.A.3	Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.	**Fractions** • Foundations • Fractions • Equivalence
3.G.A.2	Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.	
GRAPH POINTS ON THE COORDINATE PLANE TO SOLVE REAL-WORLD AND MATHEMATICAL PROBLEMS		
5.G.A.2	Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	**Problem Solving** • Problem Solving
SOLVE REAL-WORLD AND MATHEMATICAL PROBLEMS INVOLVING AREA, SURFACE AREA, AND VOLUME		
6.G.A.1	Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.	**Problem Solving** • Problem Solving
6.G.A.2	Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = l w h and V = b h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.	

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COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS		MONTESSORI CHAPTERS AND SECTIONS
6.G.A.4	Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	**Problem Solving** • Problem Solving
NUMBERS AND OPERATIONS IN BASE 10 (NBT)		
EXTEND THE COUNTING SEQUENCE		
1.NBT.A.1	Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.	**The Decimal System** • Number and Quantity
UNDERSTAND PLACE VALUE		
1.NBT.B.2	Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:	**The Decimal System** • Place Value
1.NBT.B.2.A	10 can be thought of as a bundle of ten ones — called a "ten."	
1.NBT.B.2,B	The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.	
1.NBT.B.2.C	The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).	
1.NBT.B.3	Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.	
2.NBT.A.1	Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:	
2.NBT.A.1.A	100 can be thought of as a bundle of ten tens— called a "hundred."	
2.NBT.A.1.B	The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).	

continues

COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS		MONTESSORI CHAPTERS AND SECTIONS
2.NBT.A.2	Count within 1000; skip-count by 5s, 10s, and 100s.	**The Decimal System** • Number and Quantity
2.NBT.A.3	Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.	**The Decimal System** • Number and Quantity • The Decimal System • Place Value
2.NBT.A.4	Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.	**The Decimal System** • Place Value
USE PLACE VALUE UNDERSTANDING AND PROPERTIES OF OPERATIONS TO ADD AND SUBTRACT		
1.NBT.C.4	Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.	**Whole Numbers** • Operations: Addition • Operations: Subtraction
1.NBT.C.5	Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.	**The Decimal System** • Number and Quantity **Whole Numbers** • Properties of Numbers • Memorization
1.NBT.C.6	Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	**Whole Numbers** • Operations: Subtraction

continues

COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS		MONTESSORI CHAPTERS AND SECTIONS
2.NBT.B.5	Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	**Whole Numbers** • Properties of Numbers • Memorization
2.NBT.B.6	Add up to four two-digit numbers using strategies based on place value and properties of operations.	**Whole Number Operations** • Addition
2.NBT.B.7	Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.	**Whole Numbers** • Operations: Addition • Operations: Subtraction
2.NBT.B.8	Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100-900.	**Whole Numbers** • Properties of Numbers • Memorization
2.NBT.B.9	Explain why addition and subtraction strategies work, using place value and the properties of operations	**Whole Numbers** • Operations: Addition • Operations: Subtraction
USE PLACE VALUE UNDERSTANDING AND PROPERTIES OF OPERATIONS TO PERFORM MULTI-DIGIT ARITHMETIC		
3.NBT.A.1	Use place value understanding to round whole numbers to the nearest 10 or 100.	**The Decimal System** • Place Value
3.NBT.A.2	Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.	**Whole Numbers** • Properties of Numbers • Memorization
3.NBT.A.3	Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.	**Whole Numbers** • Operations: Multiplication

continues

COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS		MONTESSORI CHAPTERS AND SECTIONS
4.NBT.B.4	Fluently add and subtract multi-digit whole numbers using the standard algorithm.	**Whole Numbers** • Properties of Numbers • Memorization
4.NBT.B.5	Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	**Fractions** • Multiplication
4.NBT.B.6	Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	**Whole Numbers** • Operations: Division
GENERALIZE PLACE VALUE UNDERSTANDING FOR MULTI-DIGIT WHOLE NUMBERS		
4.NBT.A.1	Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.	**The Decimal System** • Place Value
4.NBT.A.2	Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	
4.NBT.A.3	Use place value understanding to round multi-digit whole numbers to any place.	
UNDERSTAND THE PLACE VALUE SYSTEM		
5.NBT.A.1	Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	**The Decimal System** • Place Value **Decimals** • Foundations

continues

COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS		MONTESSORI CHAPTERS AND SECTIONS
5.NBT.A.2	Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	**Decimals** • Operations
5.NBT.A.3	Read, write, and compare decimals to thousandths.	**The Decimal System** • Place Value
5.NBT.A.3.A **5.NBT.A.3.B**	Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	**Decimals** • Foundations
5.NBT.A.4	Use place value understanding to round decimals to any place.	
PERFORM OPERATIONS WITH MULTI-DIGIT WHOLE NUMBERS AND WITH DECIMALS TO HUNDREDTHS		
5.NBT.B.5	Fluently multiply multi-digit whole numbers using the standard algorithm.	**Whole Numbers** • Operations: Multiplication
5.NBT.B.6	Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	**Fractions** • Division
5.NBT.B.7	Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	**Decimals** • Operations

continues

COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS		MONTESSORI CHAPTERS AND SECTIONS
NUMBER AND OPERATIONS FRACTIONS (NF)		
DEVELOP UNDERSTANDING OF FRACTIONS AS NUMBERS		
3.NF.A.1	Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.	**Fractions** • Foundations
3.NF.A.2	Understand a fraction as a number on the number line; represent fractions on a number line diagram.	
3.NF.A.2.A	Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.	**Fractions** • Foundations
3.NF.A.2.B	Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.	
3.NF.A.3	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.	**Fractions** • Foundations • Fractions • Equivalence
3.NF.A.3.A	Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.	**Fractions** • Equivalence
3.NF.A.3.B	Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.	
3.NF.A.3.C	Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.	**Fractions** • Foundations • Fractions • Equivalence

continues

COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS		MONTESSORI CHAPTERS AND SECTIONS
3.NF.A.3.D	Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	**Fractions** • Foundations
EXTEND UNDERSTANDING OF FRACTION EQUIVALENCE AND ORDERING		
4.NF.A.1	Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.	**Fractions** • Equivalence
4.NF.A.2	Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	**Fractions** • Foundations
BUILD FRACTIONS FROM UNIT FRACTIONS		
4.NF.B.3	Understand a fraction a/b with a > 1 as a sum of fractions 1/b.	**Fractions** • Addition and Subtraction
4.NF.B.3.A	Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.	
4.NF.B.3.B	Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.	

continues

COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS		MONTESSORI CHAPTERS AND SECTIONS
4.NF.B.3.C	Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.	**Fractions** • Addition and Subtraction
4.NF.B.3.D	Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.	**Fractions** • Addition and Subtraction **Problem Solving** • Problem Solving
4.NF.B.4	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.	**Fractions** • Multiplication
4.NF.B.4.A	Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).	
4.NF.B.4.B	Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)	
4.NF.B.4.C	Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?	**Problem Solving** • Problem Solving **Fractions** • Multiplication

continues

COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS		MONTESSORI CHAPTERS AND SECTIONS
UNDERSTAND DECIMAL NOTATION FOR FRACTIONS, AND COMPARE DECIMAL FRACTIONS		
4.NF.C.5	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.2 For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.	**Fractions** • Equivalence • Fractions • Addition and Subtraction
4.NF.C.6	Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.	**Decimals** • Foundations
4.NF.C.7	Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.	
USE EQUIVALENT FRACTIONS AS A STRATEGY TO ADD AND SUBTRACT FRACTIONS		
5.NF.A.1	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)	**Fractions** • Addition and Subtraction **Decimals** • Foundations
5.NF.A.2	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.	

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COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS		MONTESSORI CHAPTERS AND SECTIONS
APPLY AND EXTEND PREVIOUS UNDERSTANDINGS OF MULTIPLICATION AND DIVISION		
5.NF.B.3	Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?	**Fractions** • Division
5.NF.B.4	Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.	**Fractions** • Multiplication
5.NF.B.4.A	Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = (ac)/(bd).	**Fractions** • Multiplication • Division
5.NF.B.4.B	Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	**Fractions** • Multiplication
5.NF.B.5	Interpret multiplication as scaling (resizing), by:	

continues

COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS		MONTESSORI CHAPTERS AND SECTIONS
5.NF.B.5.A	Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.	**Decimals** • Operations **Whole Numbers** • Multiples, Factors, and Divisibility
5.NF.B.5.B	Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence a/b = (n × a)/(n × b) to the effect of multiplying a/b by 1.	**Fractions** • Multiplication
5.NF.B.6	Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.	**Problem Solving** • Problem Solving **Fractions** • Multiplication
5.NF.B.7	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.1	**Fractions** • Division
5.NF.B.7.A	Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.	
5.NF.B.7.B	Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.	

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COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS		MONTESSORI CHAPTERS AND SECTIONS
5.NF.B.7.C	Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?	**Problem Solving** • Problem Solving **Fractions** • Division
EXPRESSIONS AND EQUATIONS (EE)		
APPLY AND EXTEND PREVIOUS UNDERSTANDINGS OF ARITHMETIC TO ALGEBRAIC EXPRESSIONS		
6.EE.A.1	Write and evaluate numerical expressions involving whole-number exponents.	**Algebra** • Algebra
6.EE.A.2	Write, read, and evaluate expressions in which letters stand for numbers.	
6.EE.A.2.	Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation "Subtract y from 5" as 5 - y.	
6.EE.A.2.	Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.	
6.EE.A.2.	Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas V = s3 and A = 6 s2 to find the volume and surface area of a cube with sides of length s = 1/2.	

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COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS		MONTESSORI CHAPTERS AND SECTIONS
6.EE.A.3	Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.	**Whole Numbers** • Properties of Numbers **Algebra** • Algebra
6.EE.A.4	Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for..	**Algebra** • Algebra
REASON ABOUT AND SOLVE ONE-VARIABLE EQUATIONS AND INEQUALITIES		
6.EE.B.5	Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	**Algebra** • Algebra
6.EE.B.6	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	
6.EE.B.7	Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q and x are all nonnegative rational numbers.	**Algebra** • Algebra **Problem Solving** • Problem Solving
6.EE.B.8	Write an inequality of the form x > c or x < c to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form x > c or x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.	

continues

COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS		MONTESSORI CHAPTERS AND SECTIONS
REPRESENT AND ANALYZE QUANTITATIVE RELATIONSHIPS BETWEEN DEPENDENT AND INDEPENDENT VARIABLES		
6.EE.C.9	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.	**Algebra** • Algebra **Problem Solving** • Problem Solving
THE NUMBER SYSTEM (NS)		
APPLY AND EXTEND PREVIOUS UNDERSTANDINGS OF MULTIPLICATION AND DIVISION TO DIVIDE FRACTIONS BY FRACTIONS		
6.NS.A.1	Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?.	**Whole Numbers** • Operations: Division

continues

COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS		MONTESSORI CHAPTERS AND SECTIONS
COMPUTE FLUENTLY WITH MULTI-DIGIT NUMBERS AND FIND COMMON FACTORS AND MULTIPLES		
6.NS.B.2	Fluently divide multi-digit numbers using the standard algorithm.	**Whole Numbers** • Properties of Numbers • Memorization
6.NS.B.3	Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.	**Decimals** • Operations
6.NS.B.4	Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2).	**Whole Numbers** • Properties of Numbers **Decimals** • Operations **Whole Numbers** • Multiples, Factors, and Divisibility
APPLY AND EXTEND PREVIOUS UNDERSTANDINGS OF NUMBERS TO THE SYSTEM OF RATIONAL NUMBERS		
6.NS.C.5	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	**Algebra** • Signed Numbers
6.NS.C.6.A	Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3, and that 0 is its own opposite.	
6.NS.C.6.C	Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.	
6.NS.C.7	Understand ordering and absolute value of rational numbers.	

continues

COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS		MONTESSORI CHAPTERS AND SECTIONS
6.NS.C.7.A	Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret -3 > -7 as a statement that -3 is located to the right of -7 on a number line oriented from left to right.	**Algebra** • Signed Numbers
6.NS.C.7.B	Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write -3 oC > -7 oC to express the fact that -3 oC is warmer than -7 oC.	
6.NS.C.7.C	Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of -30 dollars, write |-30| = 30 to describe the size of the debt in dollars.	
6.NS.C.7.D	Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.	
6.NS.C.8	Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	**Problem Solving** • Problem Solving
RATIONAL AND PROPORTIONAL RELATIONSHIPS (RP)		
UNDERSTAND RATIO CONCEPTS AND USE RATIO REASONING TO SOLVE PROBLEMS		
6.RP.A.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."	**Ratio, Proportion, and Percent** • Ratio, Proportion, and Percent

continues

COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS		MONTESSORI CHAPTERS AND SECTIONS
6.RP.A.2	Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."1	**Ratio, Proportion, and Percent** • Ratio, Proportion, and Percent
6.RP.A.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.	**Ratio, Proportion, and Percent** • Ratio, Proportion, and Percent **Problem Solving** • Problem Solving
6.RP.A.3.A	Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.	**Ratio, Proportion, and Percent** • Ratio, Proportion, and Percent
6.RP.A.3.B	Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?	**Ratio, Proportion, and Percent** • Ratio, Proportion, and Percent **Problem Solving** • Problem Solving
6.RP.A.3.C	Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.	
6.RP.A.3.D	Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	**Ratio, Proportion, and Percent** • Ratio, Proportion, and Percent
STATISTICS AND PROBABILITY (SP)		
DEVELOP UNDERSTANDING OF STATISTICAL VARIABILITY		
6.SP.A.1	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.	**Ratio, Proportion, and Percent** • Ratio, Proportion, and Percent **Measurement and Data** • Data and Graphs

continues

COMMON CORE STATE STANDARDS STRANDS, DIVISIONS, AND STANDARDS		MONTESSORI CHAPTERS AND SECTIONS
6.SP.A.2	Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.	**Ratio, Proportion, and Percent** • Ratio, Proportion, and Percent **Measurement and Data** • Data and Graphs
6.SP.A.3	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.	
SUMMARIZE AND DESCRIBE DISTRIBUTIONS		
6.SP.B.4	Display numerical data in plots on a number line, including dot plots, histograms, and box plots.	**Ratio, Proportion, and Percent** • Ratio, Proportion, and Percent **Measurement and Data** • Data and Graphs
6.SP.B.5	Summarize numerical data sets in relation to their context, such as by:	
6.SP.B.5.A	Reporting the number of observations.	
6.SP.B.5.B	Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.	
6.SP.B.5.C	Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.	
6.SP.B.5.D	Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.	

MONTESSORI TO STANDARDS INDEX

MONTESSORI CHAPTER AND SECTION	CCSS.MATH STANDARDS AIGNED	
MATHEMATICAL PRACTICE		
Human Tendencies and the Mathematical Mind	**MP**	**No Standards aligned**
Problem Solving		**Standards for Mathematical Practice** • Problem Solving
Communication: Reasoning and Explaining		**Standards for Mathematical Practice** • Communication: Reasoning and Explaining
Modelling and Using Tools		**Standards for Mathematical Practice** • Modelling and Using Tools
Seeing Structures and Generalizing		**Standards for Mathematical Practice** • Seeing Structures and Generalizing
FOUNDATIONS		
Great Story: The Story of Numbers	**D2.His**	**History*** • Change, Continuity and Context * C3 Framework
	SL	**Speaking and Listening*** • Comprehension and Collaboration * CCSS.ELA-LITERACRY.SL
THE DECIMAL SYSTEM		
Number and Quantity	**NBT**	**Numbers and Operations in Base Ten** • Extend the counting sequence • Understand place value • Use place value understanding and properties of operations to add and subtract
	OA	**Operations and Algebraic Thinking** • Work with equal groups of objects to gain foundations for multiplication

continues

MONTESSORI CHAPTER AND SECTION	CCSS.MATH STANDARDS AIGNED	
Place Value	**NBT**	**Numbers and Operations in Base Ten** • Understand place value • Use place value understanding and properties of operations to perform multi-digit arithmetic • Generalize place value understanding for multi-digit whole numbers • Understand the place value system
WHOLE NUMBERS		
Operations: Addition	**NBT**	**Numbers and Operations in Base Ten** • Use place value understanding and properties of operations to add and subtract
	OA	**Operations and Algebraic Thinking** • Work with equal groups of objects to gain foundations for multiplication • Add and subtract within 20 • Work with addition and subtraction equations • Generate and analyze patterns • Analyze patterns and relationships • Solve problems involving the four operations and identify and explain patterns in arithmetic • Use the four operations with whole numbers to solve problems • Represent and solve problems involving addition and subtraction

continues

MONTESSORI CHAPTER AND SECTION	CCSS.MATH STANDARDS AIGNED	
Operations: Subtraction	**NBT**	**Numbers and Operations in Base Ten** • Use place value understanding and properties of operations to add and subtract
	OA	**Operations and Algebraic Thinking** • Add and subtract within 20 • Work with addition and subtraction equations • Generate and analyze patterns • Analyze patterns and relationships • Solve problems involving the four operations and identify and explain patterns in arithmetic • Use the four operations with whole numbers to solve problems • Represent and solve problems involving addition and subtraction • Understand and apply properties of operations and the relationship between addition and subtraction
Operations: Multiplication	**NBT**	**Numbers and Operations in Base Ten** • Use place value understanding and properties of operations to perform multi-digit arithmetic • Perform operations with multi-digit whole numbers and with decimals to hundredths • Use place value understanding and properties of operations to add and subtract
	OA	**Operations and Algebraic Thinking** • Generate and analyze patterns • Analyze patterns and relationships • Solve problems involving the four operations and identify and explain patterns in arithmetic • Use the four operations with whole numbers to solve problems • Represent and solve problems involving multiplication and division • Multiply and divide within 100

continues

MONTESSORI CHAPTER AND SECTION	CCSS.MATH STANDARDS AIGNED	
Operations: Division	**NBT**	**Numbers and Operations in Base Ten** • Use place value understanding and properties of operations to perform multi-digit arithmetic
	OA	**Operations and Algebraic Thinking** • Generate and analyze patterns • Analyze patterns and relationships • Solve problems involving the four operations and identify and explain patterns in arithmetic • Use the four operations with whole numbers to solve problems • Apply and extend previous understandings of multiplication and division to divide fractions by fractions • Represent and solve problems involving multiplication and division • Understand properties of multiplication and the relationship between multiplication and division • Multiply and divide within 100
	NS	**The Number System** • Apply and extend previous understandings of multiplication and division to divide fractions by fractions
Properties of Numbers	**NBT**	**Numbers and Operations in Base Ten** • Use place value understanding and properties of operations to add and subtract • Use place value understanding and properties of operations to perform multi-digit arithmetic
	OA	**Operations and Algebraic Thinking** • Solve problems involving the four operations and identify and explain patterns in arithmetic • Multiply and divide within 100 • Understand and apply properties of operations and the relationship between addition and subtraction • Add and subtract within 20 • Understand properties of multiplication and the relationship between multiplication and division2 • Compute fluently with multi-digit numbers and find common factors and multiples

continues

MONTESSORI CHAPTER AND SECTION	CCSS.MATH STANDARDS AIGNED	
Properties of Numbers	**EE**	**Equations and Expressions** • Apply and extend previous understandings of arithmetic to algebraic expressions
	NS	**The Number System** • Apply and extend previous understandings of multiplication and division to divide fractions by fractions
Memorization	**SL**	**Numbers and Operations in Base Ten** • Use place value understanding and properties of operations to add and subtract • Use place value understanding and properties of operations to perform multi-digit arithmetic
	NS	**The Number System** • Compute fluently with multi-digit numbers and find common factors and multiples
Multiples, Factors, and Divisibility	**OA**	**Operations and Algebraic Thinking** • Gain familiarity with factors and multiples
	NF	**Number and Operations Fractions** • Apply and extend previous understandings of multiplication and division
	NS	**The Number System** • Compute fluently with multi-digit numbers and find common factors and multiples
FRACTIONS		
Foundations	**G**	**Geometry** • Reason with shapes and their attributes
	NF	**Number and Operations Fractions** • Develop understanding of fractions as numbers • Extend understanding of fraction equivalence and ordering

continues

MONTESSORI CHAPTER AND SECTION	CCSS.MATH STANDARDS AIGNED	
Equivalence	**G**	**Geometry** • Reason with shapes and their attributes
	NF	**Number and Operations Fractions** • Develop understanding of fractions as numbers • Extend understanding of fraction equivalence and ordering • Understand decimal notation for fractions and compare decimal fractions
Addition and Subtraction	**NF**	**Number and Operations Fractions** • Build fractions from unit fractions • Use equivalent fractions as a strategy to add and subtract fractions • Understand decimal notation for fractions and compare decimal fractions
Multiplication	**NBT**	**Numbers and Operations in Base Ten** • Use place value understanding and properties of operations to perform multi-digit arithmetic
	NF	**Number and Operations Fractions** • Build fractions from unit fractions • Apply and extend previous understandings of multiplication and division
Division	**NBT**	**Numbers and Operations in Base Ten** • Perform operations with multi-digit whole numbers and with decimals to hundredths
	NF	**Number and Operations Fractions** • Apply and extend previous understandings of multiplication and division

continues

MONTESSORI CHAPTER AND SECTION	CCSS.MATH STANDARDS AIGNED	
DECIMALS		
Foundations	**NBT**	**Numbers and Operations in Base Ten** • Understand the place value system
	NF	**Number and Operations Fractions** • Use equivalent fractions as a strategy to add and subtract fractions • Understand decimal notation for fractions and compare decimal fractions
Operations	**NBT**	**Numbers and Operations in Base Ten** • Understand the place value system • Perform operations with multi-digit whole numbers and with decimals to hundredths
	OA	**Operations and Algebraic Thinking** • Apply and extend previous understandings of multiplication and division
	NS	**The Number System** • Compute fluently with multi-digit numbers and find common factors and multiples
ALGEBRA		
Powers of Numbers		**Standards for exponents begin in 8th grade.**
Signed Numbers	**NS**	**The Number System** • Apply and extend previous understandings of numbers to the system of rational numbers
Algebra	**OA**	**Operations and Algebraic Thinking** • Write and interpret numerical expressions
	EE	**Expressions and Equations** • Apply and extend previous understandings of arithmetic to algebraic expressions • Reason about and solve one-variable equations and inequalities • Represent and analyze quantitative relationships between dependent and independent variables

continues

MONTESSORI CHAPTER AND SECTION	CCSS.MATH STANDARDS AIGNED	
PROBLEM SOLVING		
Problem Solving	**RP**	**Ratio and Proportion** • Understand ratio concepts and use ratio reasoning to solve problems
	NF	**Number and Operations Fractions** • Build fractions from unit fractions • Use equivalent fractions as a strategy to add and subtract fractions • Apply and extend previous understandings of multiplication and division,
	OA	**Operations and Algebraic Thinking** • Solve problems involving the four operations, and identify and explain patterns in arithmetic • Use the four operations with whole numbers to solve problems • Represent and solve problems involving addition and subtraction • Represent and solve problems involving multiplication and division
	EE	**Expressions and Equations** • Reason about and solve one-variable equations and inequalities • Represent and analyze quantitative relationships between dependent and independent variables,
	NS	**The Number System** • Apply and extend previous understandings of numbers to the system of rational numbers,
	G	**Geometry** • Graph points on the coordinate plane to solve real-world and mathematical problems • Solve real-world and mathematical problems involving area, surface area, and volume

continues

MONTESSORI CHAPTER AND SECTION	CCSS.MATH STANDARDS AIGNED	
Problem Solving	**MD**	**Measurement and Data** • Work with time and money • Relate addition and subtraction to length • Solve problems involving measurement and estimation • Solve problems involving measurement and conversion of measurements • Geometric measurement: recognize perimeter • Geometric measurement: understand concepts of volume
RATIO, PROPORTION, AND PERCENT		
Ratio, Proportion, and Percent	**MD**	**Measurement and Data** • Represent and interpret data
	SP	**Statistics and Probability** • Develop understanding of statistical variability • Summarize and describe distributions
	RP	**Ratio and Proportion** • Understand ratio concepts and use ratio reasoning to solve problems
MEASUREMENT AND DATA		
Data and Graphs	**MD**	**Measurement and Data** • Represent and interpret data
	SP	**Statistics and Probability** • Develop understanding of statistical variability • Summarize and describe distributions
Measurement	**MD**	**Measurement and Data** • Measure lengths indirectly and by iterating length units, • Tell and write time, • Work with time and money, • Measure and estimate lengths in standard units, • Relate addition and subtraction to length, • Solve problems involving measurement and estimation, • Solve problems involving measurement and conversion of measurements, • Convert like measurement units within a given measurement system